The Effects of Trade Policy Uncertainty on Exporters and Multinational Firms

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Abstract

Recently, there has been a large increase in trade policy uncertainty, while multinational firms have become increasingly important in the economy. The trade policy uncertainty literature, however, has limited its attention to models with exporters only, thus abstracting from multinational firms. This paper builds a two-country DSGE model where firms can choose to serve their domestic market only, or to also sell to the foreign country either as an exporter or by engaging in foreign direct investment and operating as a multinational firm (MNE). The model features endogenous exporting and MNE entry and exit, where firms need to pay per-period entry or continuation cost. I show that uncertainty about non-tariff measures lead to quantitatively different effects on the economy when multinational firms are taken into consideration, with the short run effects of trade policy uncertainty closer to the higher end.

Keywords: trade policy uncertainty, tariffs, uncertainty shocks, multinational firm, international trade

JEL Classifications: D80, F13, F23

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1 Introduction

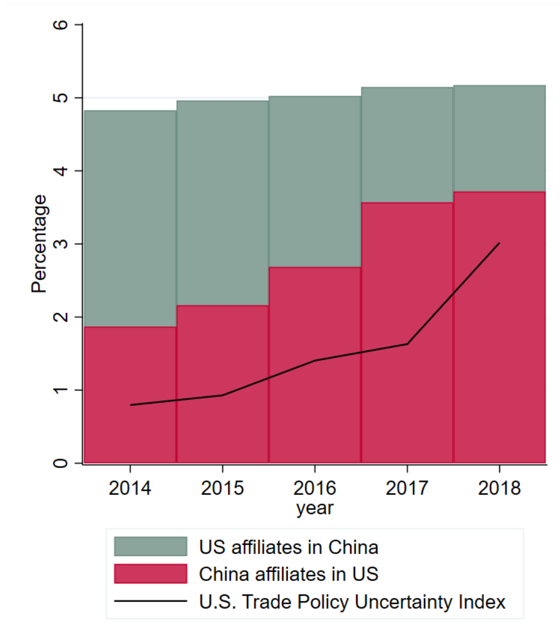
Uncertainty surrounding Brexit and the ongoing U.S.-China trade war have led to trade policy uncertainty reaching a historic high in recent years, as demonstrated by the newspaper-based U.S. Trade Policy Uncertainty index first developed by Baker et al. (2016) and recently revisited by Hong (2021). However, theoretical papers examining the effects of trade policy uncertainty have only started to emerge in recent years. Among these, there have been opposing conclusions on both the directions and the importance of the effects, with Caldara et al. (2020) showing that import tariff uncertainty will lead to a persistent fall in the components of GDP, but also an increase in export participation. On the other hand, research focusing on Brexit uncertainty costs such as Steinberg (2019) showed that while Brexit uncertainty costs are similar to the order of magnitude of the cost of business cycles, it is much smaller compared to the costs of Brexit taking place.

More importantly, models in the literature have thus far abstracted from multinational firms, which is an alternative way for firms to serve a foreign market. In fact, there is empirical evidence that global sales from multinational firms are around 2 times the sales from exports (Antràs and Yeaple, 2014). Moreover, even though trade policy uncertainty between U.S. and China has been increasing in recent years (Hong, 2021), data from the U.S. Bureau of Economic Analysis indicates that the number of affiliates that U.S. companies operate in China, as well as the number of affiliates that Chinese companies operate in the U.S. have been increasing steadily, as shown in Figure 1.

This paper thus contributes to the growing literature by building a two-country Dynamic Stochastic General Equilibrium (DSGE) model that features a continuum of monopolistically competitive firms that can choose to (i) serve solely the domestic market, or (ii) also serve the foreign market as exporters; or (iii) operate as multinational firms (MNEs) with both a domestic and a foreign plant.

Multinational firms are modeled such that the parent and its foreign affiliate share some production inputs, as in McGrattan and Prescott (2010), Kapička (2012) and Anagnostopoulos et al. (2019). Specifically, where each plant uses physical (or tangible) capital and labour hired from the local market, but the technological capital can be shared across plants owned by the same parent firm. This type of technological capital reflects the technological know-how accumulated by firms and is made up of research and development investment, branding investment and organizational capital. This paper assumes firms have three modes of operation – production for the domestic market only, production for the foreign market as well through exporting or instead serving the

Figure 1: **The mass of MNE affiliates between US and China**



Notes: The mass of US affiliates in China is presented as a percentage of the total US affiliates in the world. The mass of Chinese affiliates in the US as a percentage of the total affiliates from the world to the US. The U.S. TPU index is from Hong (2021), while data on affiliates is from the U.S. Bureau of Economic Analysis.

foreign market as MNE by operating a foreign affiliate. Gumpert et al. (2020) showed that this is a reasonable assumption using French and Norwegian firm-level data, as firms usually serve a foreign country either mainly as an exporter or mainly as a multinational firm. This present paper also assumes that firms become MNEs via greenfield entry instead of mergers and acquisitions. Moreover, firms can only engage in horizontal foreign direct investment, where they expand by replicating their production overseas to save on trade costs.

Firms need to pay a per-period cost that depends on the firm's state (either local, export, or MNE) in the beginning of the period, with entry costs higher than continuation costs à la Alessandria and Choi (2007). Whereas Alessandria and Choi (2007) only focused on export entry and continuation costs as firms in their model were only allowed to choose between staying local or exporting, this paper also includes MNE entry and continuation costs. This feature thus allows the model to incorporate endogenous export and MNE entry and exit decisions.

In this paper, I consider uncertainty about non-tariff barriers to trade instead of uncertainty about import tariffs. International trade has been relatively liberalized due to the General Agreement on Tariffs and Trade (GATT) and WTO, as well as preferential trade agreements (PTA) and customs unions. Thus, non-tariff barriers often play a bigger role than tariffs, as shown in studies such as Anderson and van Wincoop (2004). These non-tariff barriers can be modeled as iceberg transport costs (e.g. Steinberg (2019), Gumpert et al. (2020)). Recent papers such as Caldara et al. (2020), when modeling trade policy uncertainty, posit that import tariffs follow an AR(1) process, with the stochastic volatility following another AR(1) process. This is unrealistic as changes in tariff levels should be more discrete. Hence, I take a more natural approach by modeling uncertainty about non-tariff measures instead as an AR(1) with AR(1) process.

As a result, this paper serves as the first attempt to study the effects of trade policy uncertainty in a model with both exporters and multinational firms. It shows that the quantitative results of trade policy uncertainty are closer to the upper bound found in the current literature, thus emphasizing the importance in including multinational firms when trying to quantify the effects of trade policy uncertainty.

The paper is organized as follows. Section 2 explains the trade-off between being an exporter and being a multinational firm. 3 provides a literature review. Section 4 details the model. Section 5 presents the solution method and Section 6 discusses the results. Section 7 concludes.

2 Trade-off between Exporters and MNEs

Consider a simple setting where an exporter (E) can choose to exit the foreign market and become a local (L) firm to earn $\Pi^L(z)$, or pay a continuation fixed cost $f^E(E)$ to earn $\Pi^E(z)$ from being an exporter. They also have the option to pay an entry fixed cost $f^M(E)$ to become a multinational firm, earning $\Pi^M(z)$. On the other hand, multinational firms (M) can pay a re-entry fixed cost $f^M(M)$ to keep the foreign affiliate open and earn $\Pi^M(z)$, or re-enter as an exporter, paying fixed cost $f^E(E)$ and earning $\Pi^E(z)$. Note that a multinational firm cannot exit completely to be a local firm. This setup is similar to the one in Helpman et al. (2004), yet with entry and continuation costs that depend on the firm's initial status. In particular, let's consider entry costs that are higher than continuation costs à la Alessandria and Choi (2007), i.e. $f^M(E) > f^M(M)$.

First consider a static trade policy at τ^L. Each period, there is a draw of productivity z. For existing exporting firms, they will find re-entering as an exporter profitable if the *additional* profit from being an exporter, relative to being a local firm, is positive:

$$\Pi^E(z) - \Pi^L(z) - f^E(E) > 0 \tag{1}$$

Thus, there exists a productivity threshold $\bar{z}^E$ that separates exporters from local firms.

Otherwise, exporters will become multinational firms if the *additional* benefit from setting a foreign plant (relative to staying local) is (i) positive:

$$\Pi^M(z) - \Pi^L(z) - f^M(E) > 0 \tag{2}$$

and (ii) higher than the *additional* benefit from re-entering as an exporter:

$$\Pi^M(z) - \Pi^E(z) - \left[f^M(E) - f^E(E)\right] > 0.$$

Given that exporters need to pay import tariff per unit of the product, the slope of Equation 2 will be lower than that of Equation 1. Thus, there exists a productivity threshold $\bar{z}^M$ where the two equation crosses that distinguishes between experienced exporters and new multinational firm entrants.

Assumption 1 *The entry fixed cost to become a MNE is greater than the export continuation cost: $f^M(E) > f^E(E)$.*

With the above assumption, as long as the tariff τ is not too large, the productivity needed to enter as a MNE is greater than the productivity needed to re-enter as an exporter: $\hat{z}^E > \bar{z}^E$.

Turning to multinational firms, they will choose to stay as a multinational firm if the *additional* profit from opening a foreign plant *relative to* shipping to the foreign market is positive:

$$\Pi^M(z) - \Pi^E(z) - \left[f^M(M) - f^E(E)\right] > 0.$$

To allow for easier graphical presentation, I decompose the multinational firm condition by evaluating the benefits from being a MNE with respect to being a local firm, i.e. combining Equation 1 with

$$\Pi^M(z) - \Pi^L(z) - f^M(M) > 0. \tag{3}$$

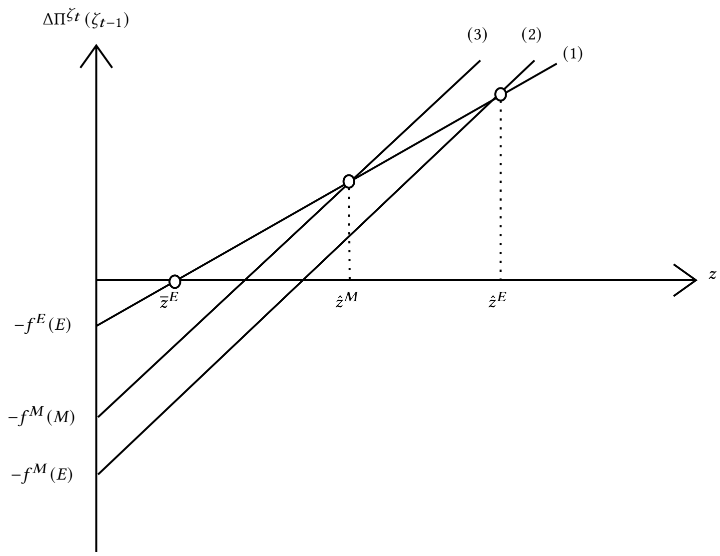
The productivity threshold where these two intersect, $\hat{z}^M$, is the threshold between MNE returning to being exporters, and re-entering as a MNE.

Assumption 2 *The continuation cost for a MNE is higher than the continuation cost for an exporter: $f^M(M) > f^E(E)$.*

Assumption 3 *The MNE entry cost is higher than the MNE continuation cost: $f^M(E) > f^M(M)$.*

With the above two conditions, the productivity threshold for a MNE re-entry $\hat{z}^M$ is between the export re-entry $\bar{z}^E$ and the MNE greenfield entry $\hat{z}^E$.

Figure 2: Trade-off between Exporters and MNEs



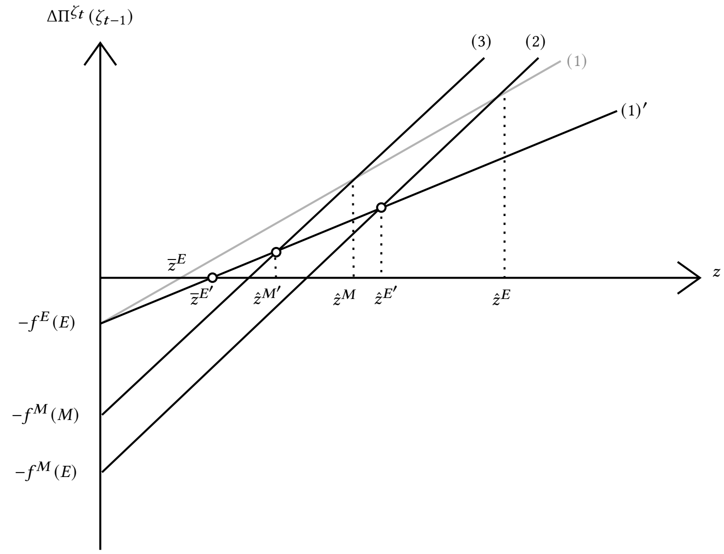
(1) $\Pi^E(z) - \Pi^L(z) - f^E(E) > 0$; (2) $\Pi^M(z) - \Pi^L(z) - f^M(E) > 0$; (3) $\Pi^M(z) - \Pi^L(z) - f^M(M) > 0$

The conditions are plotted in Figure 2. The intercepts indicate the fixed entry or continuation costs, while the flatter slope from Equation 1, i.e. the additional profit from being an exporter, is due to the per unit tariff.

Now, if the tariff level is increased to τ^H, the benefit from exporting is decreased, thus flattening the curve for Equation 1. The illustration is shown in Figure 3. As seen, the productivity threshold $\bar{z}^E$ for firms to re-enter as an exporter, relative to exiting, has increased. This means that it is more likely for firms to become local firms. On the other hand, the threshold for exporters to consider being a multinational firm $\hat{z}^E$ has decreased, as well as the threshold for MNEs to remain as is, $\hat{z}^M$. This leads to an increase in the mass of MNEs.

In a world where firms' decisions are dynamic, such as making irreversible and costly investments, or that there are price adjustment cost, the decisions to switch between exporters and multinational firms under higher tariff level will not be this clear-cut. However, this establishes the idea that increases in tariffs level can lead to an increase in the mass of MNEs, either due to exporters switching to become a MNE, or as MNEs staying on as MNEs. When tariffs are uncertain, this transition will be depend on multiple factors, such as the nature and persistence of the uncertainty, the exact trade-off that the firms face between being an exporter and being a MNE, and the rigidities present in the economy. In this paper, I thus build a RBC model that is

Figure 3: Trade-off between Exporters and MNEs: Higher Tariff



Notes: This figure presents the scenario if the per unit tariff is increased from τ^L to τ^H. All the new equations and productivity thresholds are denoted with a prime. See Figure 2 for the original equations.

consistent with the salient MNE features observed in the economy. I now first discuss the present literature.

3 Literature Review

This paper combines three strands of the literature – uncertainty, trade (exporting and multinational firms), and trade policy uncertainty – to contribute to the growing literature on the effects of trade policy uncertainty. In this section, I will review each strand in turn.

While uncertainty has long been studied since Bernanke (1983) and Dixit and Pindyck (1994), there has been a rise in papers studying uncertainty shocks in a general equilibrium framework in recent years. These papers have evolved from the usual General Autoregressive Conditional Heteroskedasticity (GARCH) models to stochastic volatility models, as the latter allow for the separation between innovations to the level and innovations to the volatility.[1] For example, the small open economy business cycle model in Fernández-Villaverde et al. (2011) assumed that the international risk-free real interest rate and the country spread both follow AR(1) processes, while their volatilities follow another two AR(1) processes. The paper showed that under an increase in

[1]The volatility in GARCH models is dependent on the past level of shocks.

country spread volatility, the household reduces its risky foreign debt holdings by lowering both consumption and investment, thus decreasing domestic absorption.

To explain the Vector Autoregression (VAR) evidence that there is a comovement between output, consumption, and investment under an uncertainty (proxied using VXO) shock, Basu and Bundick (2017) considered a stochastic discount rate with stochastic volatility. The paper showed that sticky prices are important for the transmission of uncertainty shocks as the comovement only exists under nominal rigidities. This is because output decreases following a reduction in consumption due to greater uncertainty. As a result, the marginal return of holding capital falls, which translates to a lower investment level. Bonciani and Oh (2019) allowed firms' total factor productivity (TFP) to have an exogenous component that follows an AR(1) process with stochastic volatility. Their DSGE model assumes Epstein-Zin preferences, endogenous growth in R&D investment, and sticky prices, all of which were found to be important to capture the long-term fall in consumption, investment, and output. The various applications of stochastic volatility models thus suggest that they will be useful in modeling trade policy uncertainty.

The next building block of this paper relies on models of international trade and multinational firms. Trade models have gone through a long history of development, and different reasons to explain trade patterns have been proposed, ranging from technological differences and comparative advantages (Ricardo, 1821), to factor endowment differences (Heckscher-Ohlin and Vanek (1968)), to increasing returns to scale (Krugman, 1979) or increased product variety (Krugman, 1980).

As it is unrealistic to assume all firms have identical technologies, the trade literature began to model heterogeneous firms. Eaton and Kortum (2002) considered heterogeneous firms producing with constant returns to scale and perfect competition. With constant elasticity of substitution (CES) preferences and productivity drawn from a Fréchet distribution, the paper shows that every country must be the lowest cost producer for some goods, and thus would be a sole exporter of those goods to all countries. Melitz (2003) analyzed international trade with heterogeneous firms that enter and exit à la Hopenhayn (1992), but with monopolistic competition instead of perfect competition. Using a "zero cutoff profit" condition, which states that firms only produce under non-negative profits, and a free export entry condition, which implies that the expected value of exporting for new entrants net of a fixed sunk cost is zero, the model shows that the least efficient firms would exit and efficient firms would increase their production. Chaney (2008) obtained closed-form solutions to Melitz (2003) by assuming that the productivity of the firms follows a Pareto distribution.

Alessandria and Choi (2007) considered endogenous export entry and exit decisions in a general equilibrium open economy business cycle model, where firms pay an export entry sunk cost and a smaller per period continuation cost in order to operate in the foreign market. Comparing their model with one that has neither export entry nor continuation costs, and with one that has no export entry cost but a stochastic continuation cost, the paper showed that the type of costs leads to very different export participation responses to a persistent positive home productivity shock. Another feature found to be important for the transmission of shocks is firms' capital accumulation, which depends on their export status. As Alessandria and Choi (2007)'s model only featured export decisions, this paper extends their framework by incorporating endogenous MNE entry and exit decisions based on imposing MNE entry and continuation cost.

The literature on multinational firms is thoroughly reviewed in Antràs and Yeaple (2014), which surveyed different types of multinational firm activity in a framework that features Krugman (1980)'s CES preferences and Melitz (2003)'s firm heterogeneity. Closest to this paper's assumptions on multinational firms (i.e. greenfield entry and horizontal expansion), Helpman et al. (2004) proposed a static partial equilibrium framework where a continuum of firms pay fixed costs (that are independent of their status in the previous period) to become a local firm or an exporter or a multinational firm. As exporters face iceberg transport costs while multinational firms face higher fixed cost, this "proximity-concentration trade-off" means that only the most productive firms serve the foreign market, and among them, the more productive ones engage in foreign direct investment. Gumpert et al. (2020) introduced dynamics to this framework by allowing firm's productivity to follow a Markov process and imposing a sunk entry cost for multinational firms. While the three-tier result from the static version is still present, the sunk entry cost leads to a band of inaction, where existing MNEs with a certain productivity level will not exit, but firms with the same productivity level will not enter as MNE.

Although Gumpert et al. (2020)'s setup provides strong modeling tools in terms of the productivity cutoffs, its setup assumes that multinational firms only use labour to produce. Given the importance of capital accumulation as shown in Alessandria and Choi (2007), this paper borrows from the framework proposed by McGrattan and Prescott (2010), Kapička (2012) and Anagnostopoulos et al. (2019), in which multinational firms also use tangible capital and technological capital to produce. McGrattan and Prescott (2010) suggested that the inclusion of technological capital is necessary to capture the high rate of return for multinational firms' subsidiaries found in empirical data. Hence, the literature usually models multinational firms such that they can use technological capital across plants in different locations, but need to hire physical capital and labour from the country in which the plant is located.

Turning to the last strand of literature that this paper builds upon, research on trade policy uncertainty in recent years has started to become more popular as it becomes a more prominent phenomenon in the world. Using a dynamic partial equilibrium model with sunk export costs, Handley and Limão (2017) compared the expected value of exporting (net of sunk entry costs) with the expected value of waiting. This model aimed at studying the impact of uncertainty about a foreign country's tariff policy on firms' decisions to invest and export. Firms experience an increased option value of waiting if uncertainty increases. This model is then applied to analyze Portugal's accession into the European Union. Graziano et al. (2018) used a similar framework to study Brexit uncertainty by considering uncertainty in both demand and tariff, showing that both trade flows and export entry participation would decrease. However, their framework assumed that firms face an exogenous death shock.

Steinberg (2019) modeled Brexit uncertainty with a three-country DSGE model, where heterogeneous firms face extensive margins à la Melitz (2003), and intensive margins à la Arkolakis (2010). Hence, firms not only have to make the export participation decision, but also need to advertise in order to penetrate into the foreign market. The model featured uncertain import tariffs and iceberg trade costs, calibrated to different Brexit scenarios. With this firm-level dynamic export participation model, Steinberg (2019) showed that while there are no substantial impacts to macroeconomic variables in the short run, Brexit will lower GDP, consumption and trade flows in the long run. Uncertainty cost is much smaller than pure Brexit effects, but has the same order of magnitude as a business cycle. The intensive margin is not found to be important for Brexit welfare costs while export participation matters. Neither has an effect on Brexit uncertainty costs.

Closest to this paper, Caldara et al. (2020) uses a two-country DSGE model where the representative household consumes a composite final good that consists of domestic and imported bundles to study the transmission of trade policy uncertainty. Trade policy uncertainty is modeled as import tariffs uncertainty, borrowing from the uncertainty literature by positing that import tariffs follow an AR(1) process with an AR(1) volatility process. However, tariffs levels are discrete and uncertainty about import tariffs can be better modeled in a Markov-switching DSGE mode. Monopolistically competitive firms face endogenous export entry and exit à la Alessandria and Choi (2007) with per-period export cost. The authors find that due to the export entry cost being higher than the export continuation cost, the probability of exporters staying as exporters would increase following import tariffs uncertainty.

Different papers have opposing conclusions on the effects of trade policy uncertainty, with Caldara et al. (2020) and Graziano et al. (2018) finding trade policy uncertainty matters whilst having opposite conclusions on the qualitative effects on export participation; and Steinberg (2019) finding minimal TPU uncertainty costs, it is important to understand the puzzle amid the ongoing trade policy uncertainty. More importantly, the trade policy uncertainty literature has thus far abstracted from models that feature multinational firms, which have been shown in empirical data to have a growing contribution to sales. Moreover, studies like Dhingra et al. (2016) and Pain and Young (2004) have explored the effects of Brexit on foreign direct investment. Foreign direct investment (FDI), where firms from other countries invest in the domestic market through greenfield investment, expansion of subsidiaries or cross-border merger and acquisitions (M&As), can be beneficial for the country due to spillover of knowledge and thus the increased productivity of the domestic market.

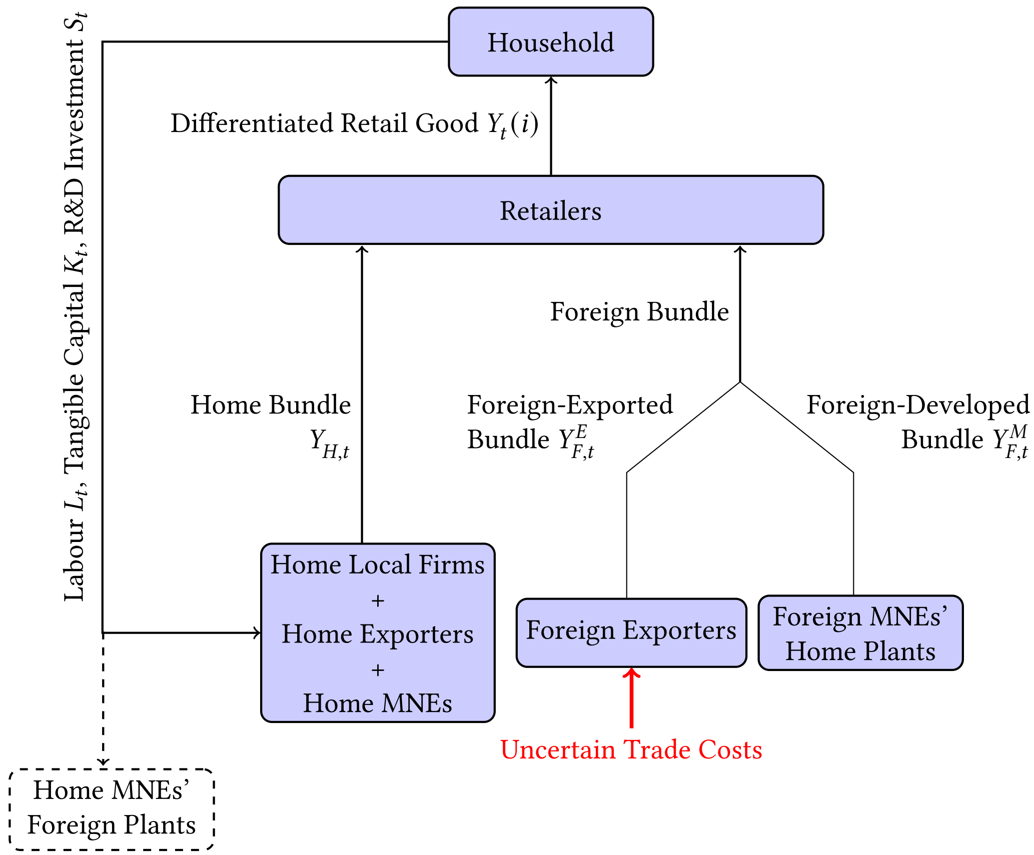
This paper contributes to the growing literature on trade policy uncertainty by considering its effects in a two-country DSGE model with both exporters and multinational firms. This is done by (i) applying the stochastic volatility models from the uncertainty literature to trade policy uncertainty, (ii) extending the endogenous export entry and exit framework from Alessandria and Choi (2007) to include endogenous MNE entry and exit, and (iii) borrowing insights from the current trade policy uncertainty research that focuses on exporters only. This paper will thus address the disagreements about trade policy uncertainty effects among the current literature via this novel extension.

4 Model

This paper considers a New-Keynesian Dynamic Stochastic General Equilibrium model with 2 countries – Home (H) and Foreign (F). The representative household consumes a composite of monopolistically competitive retail goods that consists of intermediate varieties produced by Home and Foreign monopolistically competitive heterogeneous firms. The model features the conventional rigidities, i.e. sticky prices and capital adjustment costs, and trade policy uncertainty about non-tariff barriers.

Foreign variables are denoted by an asterisk; and the country of origin and the production location of the goods are denoted in the first and second subscript respectively. Hence, for example, $y_{H,t}$ denotes a good consumed by Home that is produced in a Home-owned Home plant at time t; and $y^*_{FH,t}$ denotes a good consumed by Foreign that is produced in a Foreign-owned Home plant at time t.

Figure 4: Model Building Blocks



Notes: The Home representative household consumes a composite of monopolistically competitive retail goods $Y_t(i)$ that consist of intermediate varieties produced by Home firms $y_{H,t}(j)$, by Foreign exporters $y^E_{F,t}(j)$, and by Foreign MNEs $y^M_{F,t}(j)$. Exporters face non-tariff measures (modeled as iceberg transport costs).

4.1 Households

Each country has a representative household that maximizes lifetime utility by choosing final good consumption C_t, hours worked L_t, nominal Home and Foreign bond holdings $B_{H,t+1}$ and $B_{F,t+1}$, and investments in physical capital I_t and technological capital S_t at time t.

The representative household has a Greenwood–Hercowitz–Huffman (GHH) utility, which is frequently used in open-economy models following Mendoza (1991), and has a habit persistence that depends on its previous consumption. Hence, it solves the following optimization problem:

$$\max_{\{C_t, L_t, I_t, S_t, B_{H,t+1}, B_{F,t+1}\}\ s.t.(5),(6)} \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \frac{\left[(C_t - \mathrm{b}\, C_{t-1}) - \frac{\psi}{1+\mu} L_t^{1+\mu} \right]^{1-\sigma}}{1-\sigma}, \tag{4}$$

where $\beta \in (0,1)$ is the subjective discount factor, $\mathrm{b} \in [0,1)$ governs the intensity of habit formation, ψ is the disutility of labour, $\mu > 0$ is the Frisch inverse elasticity parameter and $\sigma > 1$ is the risk aversion parameter.

The representative household receives the real wage rate w_t for its labour hired by intermediate firms and the real physical capital rental rate r_t^k for its physical capital K_t rented by intermediate firms. The household also receives the real price p_t^s for selling technological investment S_t to the intermediate firms, but faces quadratic investment adjustment cost à la Christiano et al. (2005). This type of technological investment is different from tangible capital and can include research and development (R&D) investment, brand equity investment, and organizational capital (Kapička, 2012). The household also receives a lump-sum transfer T_t from the government, which is in units of the final good, as well as the nominal profits from retailers $\Pi_{Y,t}(i)$ and the monopolistically competitive firms $\Pi_t(j)$. R_{t-1} is the nominal gross return on the one-period nominal Home bonds $B_{H,t}$ issued in period $t-1$ and P_t is the nominal price of the final good. Similar for R^*_{t-1}, nominal gross return on the nominal Foreign bonds $B_{F,t}$.

Hence, the budget constraint is

$$\begin{aligned} C_t + &\left[1 + \frac{\kappa}{2} \left(\frac{S_t}{S_{t-1}} - 1 \right)^2 \right] S_t + I_t + \frac{B_{H,t+1}}{P_t} + \frac{\epsilon_t B_{F,t+1}}{P_t} \\ &= w_t L_t + r_t^k K_t + p_t^s S_t + \frac{R_{t-1}}{P_t} B_{H,t} + \frac{\epsilon_t R^*_{t-1}}{P_t} B_{F,t} + T_t + \int \frac{\Pi_{Y,t}(i)}{P_t} \mathrm{d}i + \int \frac{\Pi_t(j)}{P_t} \mathrm{d}j, \end{aligned} \tag{5}$$

where $\kappa > 0$ is the technological investment adjustment parameter.

The law of motion for the physical capital investment is

$$K_{t+1} = (1-\delta_k)K_t + \left[1 - \frac{\kappa}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2\right] I_t, \tag{6}$$

where $\delta_k \in [0,1]$ is the depreciation rate.

Consumption and investments in both technology and physical capital are made up of the final good Y_t, which is a constant elasticity of substitution (CES) aggregate composite of retail goods $Y_t(i)$ with an elasticity of substitution of $\phi \geq 0$:

$$C_t + \left[1 + \frac{\kappa}{2}(\frac{S_t}{S_{t-1}} - 1)^2\right] S_t + I_t + \frac{\rho_P}{2}(\frac{P_t}{P_{t-1}} - 1)^2 Y_t = \left[\int Y_t(i)^{\frac{\phi-1}{\phi}} di\right]^{\frac{\phi}{\phi-1}} \equiv Y_t, \tag{7}$$

where the $\frac{\rho_P}{2}(\frac{P_t}{P_{t-1}} - 1)^2 Y_t$ term is from the price adjustment faced by retailers, to be defined in the next subsection.

4.2 Retailers

Each country has a continuum of monopolistically competitive retailers indexed by $i \in [0,1]$ that use a nested CES technology to produce a differentiated retail good $Y_t(i)$. In particular, the retail good is a CES bundle of the Home bundle $Y_{H,t}(i)$ and the Foreign bundle $Y_{F,t}(i)$:

$$Y_t(i) = \left[\omega^{\frac{1}{\theta}} Y_{H,t}(i)^{\frac{\theta-1}{\theta}} + (1-\omega)^{\frac{1}{\theta}}\ Y_{F,t}(i)^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}, \tag{8}$$

where $\omega \in [0,1]$ denotes the relative weight of the Home bundle (indicating Home bias for $\omega > \frac{1}{2}$), and $\theta > 0$ is the elasticity of substitution between the bundles.

The Foreign bundle is in turn a CES aggregate of the Foreign exporters bundle and the Foreign MNE bundle:

$$Y_{F,t}(i) = \left[\nu^{\frac{1}{\eta}} Y^E_{F,t}(i)^{\frac{\eta-1}{\eta}} + (1-\nu)^{\frac{1}{\eta}}\ Y^M_{F,t}(i)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}, \tag{9}$$

where $\nu \in [0,1]$ denotes the relative weight of the exported bundle, and $\eta > 0$ is the elasticity of substitution between the bundles.

The aggregate Home bundle consumed by the Home country $Y_{H,t}$ combines all Home intermediate varieties $y^{\varsigma}_{H,t}(j)$ from (i) Home local firms, (ii) Home firms that are exporters, and (iii) Home plants that are owned by Home multinational firms, where $\varsigma \in \{L, E, M\}$. The Foreign exported bundle $Y^E_{F,t}$ contains intermediate varieties $y^E_{F,t}(j)$ from Foreign exporters while the Foreign MNE bundle $Y^M_{F,t}$ contains varieties $y^M_{F,t}(j)$ from Foreign MNEs.[2] The aggregate bundles thus have the following forms:[3]

$$Y_{H,t} = \left[\int_{j\in[0,1]} y_{H,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} \mathrm{d}j\right]^{\frac{\varepsilon}{\varepsilon-1}}; \tag{10}$$

$$Y^E_{F,t} = \left[\int_{j\in\mathcal{E}^*_t} y^E_{F,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} \mathrm{d}j\right]^{\frac{\varepsilon}{\varepsilon-1}}; \tag{11}$$

$$Y^M_{F,t} = \left[\int_{j\in\mathcal{M}^*_t} y^M_{F,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} \mathrm{d}j\right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{12}$$

where $\varepsilon > 0$ is the elasticity of substitution between intermediate varieties, and $\mathcal{M}^*_t$ and $\mathcal{E}^*_t$ denote the set of Foreign multinational firms and exporters in period t, respectively.

Retailers' nominal profits are thus:

$$\Pi_{Y,t}(i) = P_t(i)Y_t(i) - P_{H,t}Y_{H,t}(i) - P_{F,t}Y_{F,t}(i) - AC_t(i), \tag{13}$$

where $P_t(i)$ is the nominal price of the differentiated retail good. $P_{H,t}$ and $P_{F,t}$, are the price indices for the Home, and Foreign bundles respectively. Retailers are subject to price adjustment costs $AC_t(i) = \frac{\rho_P}{2}\left(\frac{P_t(i)}{P_{t-1}(i)} - 1\right)^2 Y_t$ à la Rotemberg (1982), with $\rho_P > 0$ being the price adjustment parameter.

Under cost minimization, the demand functions for the bundles are

$$Y_{H,t}(i) = \omega\left[\frac{P_{H,t}}{MC_t}\right]^{-\theta} Y_t(i); \tag{14}$$

$$Y_{F,t}(i) = (1-\omega)\left[\frac{P_{F,t}}{MC_t}\right]^{-\theta} Y_t(i) \tag{15}$$

where $MC_t = \left[\omega\left(P_{H,t}\right)^{1-\theta} + (1-\omega)\left(P_{F,t}\right)^{1-\theta}\right]^{\frac{1}{1-\theta}}$ is the marginal cost of retailers.

[2] It is assumed that the household views intermediate varieties from Foreign-owned Home plants as Foreign products.

[3] Firms are symmetric under Rotemberg price adjustment.

$$Y^E_{F,t}(i) = \nu \left[\frac{e^{\tau_t} P^E_{F,t}}{P_{F,t}} \right]^{-\eta} Y^F_t(i); \tag{16}$$

$$Y^M_{F,t}(i) = (1-\nu) \left[\frac{P^M_{F,t}}{P_{F,t}} \right]^{-\eta} Y^F_t(i). \tag{17}$$

The demand functions for the intermediate varieties are

$$y^\varsigma_{H,t}(j) = \left[\frac{P^\varsigma_{H,t}(j)}{P_{H,t}} \right]^{-\varepsilon} Y_{H,t} = \omega \left[\frac{P^\varsigma_{H,t}(j)}{P_{H,t}} \right]^{-\varepsilon} \left[\frac{P_{H,t}}{MC_t} \right]^{-\theta} Y_t(i), \quad \varsigma \in \{L, E, M\}; \tag{18}$$

$$y^E_{F,t}(j) = \left[\frac{P^E_{F,t}(j)}{P^E_{F,t}} \right]^{-\varepsilon} Y^E_{F,t} = \nu\,(1-\omega) \left[\frac{P^E_{F,t}(j)}{P^E_{F,t}} \right]^{-\varepsilon} \left[\frac{e^{\tau_t} P^E_{F,t}}{P_{F,t}} \right]^{-\eta} \left[\frac{P_{F,t}}{MC_t} \right]^{-\theta} Y_t(i), \tag{19}$$

$$y^M_{F,t}(j) = \left[\frac{P^M_{F,t}(j)}{P^M_{F,t}} \right]^{-\varepsilon} Y^M_{F,t} = (1-\nu)\,(1-\omega) \left[\frac{P^M_{F,t}(j)}{P^M_{F,t}} \right]^{-\varepsilon} \left[\frac{P^M_{F,t}}{P_{F,t}} \right]^{-\eta} \left[\frac{P_{F,t}}{MC_t} \right]^{-\theta} Y_t(i); \tag{20}$$

where $P_{H,t}(j)$, $P^M_{F,t}(j)$ and $P^E_{F,t}(j)$ are the Home currency intermediate varieties prices set by Home firms, Foreign multinational firms, and Foreign exporters, respectively.

4.3 Monopolistically Competitive Firms

Each country has a continuum on the unit interval of monopolistically competitive firms that produce one intermediate variety each. Each firm j can operate in one of three different configurations indicated by ς_t: (i) $\varsigma_t = L$: operate only locally to produce $y^L_{H,t}(j)$ for the Home market, (ii) $\varsigma_t = E$: operate locally to produce $y^E_{H,t}(j)$, as well as $y^{E*}_{H,t}(j)$ for Foreign via exporting; or (iii) $\varsigma_t = M$: operate locally to produce $y^M_{H,t}(j)$, as well as $y^{M*}_{H,t}(j)$ for Foreign by operating a Foreign plant as MNE.

The production function has the usual constant-returns-to-scale Cobb-Douglas form, where $\chi \in (0,1)$ governs the input share of technological capital $n_t(j)$, $\alpha\,(1-\chi)$ governs the input share of capital $k_t(j)$ and $(1-\alpha)(1-\chi)$ governs the input share of labour $l_t(j)$, with $\alpha \in (0,1)$. Intermediate firms' total factor productivity depends on the aggregate productivity A_t, which follows an AR(1) process with persistence $\rho_A \in (0,1)$, and the idiosyncratic i.i.d. productivity $z_t(j)$ that has standard deviation $\sigma_z > 0$.

The production function for a firm that is either a local firm $j \in \mathcal{L}_t$ or an exporter $j \in \mathcal{E}_t$ is thus:

$$y^{\varsigma}_{H,t}(j) + E_t(j)\, e^{\xi_t}\, y^{E*}_{H,t}(j) = A_t z_t(j)\, n^{\varsigma-1}_{H,t}(j)^{\chi} \left(k^{\varsigma}_{H,t}(j)^{\alpha} l^{\varsigma}_{H,t}(j)^{1-\alpha}\right)^{1-\chi}, \quad \varsigma \in \{L, E\} \tag{21}$$

where $E_t(j) = \begin{cases} 1 & \text{if} \quad j \in \mathcal{E}_t \\ 0 & \text{otherwise} \end{cases}$ serves as an indicator function for whether firm j is an exporter or not at time t. If firms export, they face non-tariff measures e^{ξ_t} modeled as iceberg transportation cost. These are posited to follow an AR(1) process with volatility following another AR(1) process.

$$\xi_t = \rho_{\xi}\xi_{t-1} + \exp(\sigma_{\xi_t})\, u_{\xi_t}, \qquad u_{\xi_t} \sim N(0,1) \tag{22}$$

$$\sigma_{\xi_t} = \rho_{\sigma_{\xi}}\sigma_{\xi_{t-1}} + v_{\xi_t}, \qquad v_{\xi_t} \sim N(0,1) \tag{23}$$

where $\rho_{\xi} \in (0,1)$ and $\rho_{\sigma_{\xi}} \in (0,1)$ govern the persistence of the two AR(1) processes. u_{ξ_t} captures unexpected changes in the level of non-tariff measures, whereas v_{ξ_t} acts as an uncertainty shock.

A multinational firm $j \in \mathcal{M}_t$ uses the same technological capital $n^{\varsigma-1}_{H,t}(j)$ in both plants, but needs to hire labour and rent physical capital from the country the plant is in. Hence, it uses Home labour $l^{M}_{H,t}(j)$ and Home physical capital $k^{M}_{H,t}(j)$ to produce $y^{M}_{H,t}(j)$ for the Home country and uses Foreign labour $l^{M}_{F,t}(j)$ and Foreign physical capital $k^{M}_{F,t}(j)$ to produce $y^{M*}_{H,t}(j)$ for the Foreign market. Multinational firms do not face iceberg transportation costs when serving the Foreign market. Thus, for $j \in \mathcal{M}_t$, the production functions for the Home and Foreign plants are respectively:

$$y^{M}_{H,t}(j) = A_t z_t(j) n^{\varsigma-1}_{H,t}(j)^{\chi} \left(k^{M}_{H,t}(j)^{\alpha} l^{M}_{H,t}(j)^{1-\alpha}\right)^{1-\chi}; \tag{24}$$

$$y^{M*}_{H,t}(j) = A_t z_t(j) n^{\varsigma-1}_{H,t}(j)^{\chi} \left(k^{M}_{F,t}(j)^{\alpha} l^{M}_{F,t}(j)^{1-\alpha}\right)^{1-\chi}. \tag{25}$$

Firms accumulate technological capital $n^{\varsigma-1}_{H,t}(j)$ according to the law of motion:

$$n^{\varsigma}_{H,t+1}(j) = (1-\delta_n)\, n^{\varsigma-1}_{H,t}(j) + s^{\varsigma}_{H,t}(j), \quad \varsigma \in \{L, E, M\} \tag{26}$$

where $\delta_n \in [0,1]$ is the rate of depreciation for technological capital, and $s_{H,t}(j)$ is the technological capital investment.

Monopolistically competitive firms maximize profits by setting their own prices. Firms pay real wages w_t for Home labour hired $l_{H,t}(j)$, $Q_t w^*_t$ for any Foreign labour hired $l^{M}_{F,t}(j)$, real rental rate

r_t^k for Home physical capital rented $k_{H,t}(j)$, $Q_t r_t^{k*}$ for Foreign physical capital rented $k_{F,t}^M(j)$, and real capital price p_t^s for the technological capital invested $s_{H,t}(j)$. Q_t is the real exchange rate to be defined below. Hence, within-period real profits $\pi_t(j) = \frac{\Pi_t(j)}{P_t}$ (excluding export/MNE entry and continuation costs) for local firms, exporters, and multinational firms are, respectively:

$$\pi_t^L(j) = p_{H,t}^L(j)y_{H,t}^L(j) - w_t l_{H,t}^L(j) - r_t^k k_{H,t}^L(j) - p_t^s s_{H,t}^L(j), \quad j \in \mathcal{L}_t \tag{27a}$$

$$\pi_t^E(j) = p_{H,t}^E(j)y_{H,t}^E(j) + Q_t p_{H,t}^{E*}(j)y_{H,t}^{E*}(j) - w_t l_{H,t}^E(j) - r_t^k k_{H,t}^E(j) - p_t^s s_{H,t}^E(j), \quad j \in \mathcal{E}_t \tag{27b}$$

$$\begin{aligned}\pi_t^M(j) = p_{H,t}^M(j)y_{H,t}^M(j) - w_t l_{H,t}^M(j) - r_t^k k_{H,t}^M(j) - p_t^s s_{H,t}^M(j) \\ + Q_t p_{H,t}^{M*}(j)y_{H,t}^{M*}(j) - Q_t w_t^* l_{F,t}^M(j) - Q_t r_t^{k*} k_{F,t}^M(j), \quad j \in \mathcal{M}_t\end{aligned} \tag{27c}$$

where $p_{H,t}^{\varsigma}(j) = \frac{P_{H,t}^{\varsigma}(j)}{P_t}$ is the real price of the Home intermediate variety $y_{H,t}^{\varsigma}(j)$, expressed in terms of the Home final good price P_t. The Foreign real price of the intermediate variety $y_{H,t}^{E*}(j)$ exported by a Home exporter is $p_{H,t}^{E*}(j) = \frac{P_{H,t}^{E*}(j)}{P_t^*}$, while the Foreign real price of the intermediate variety $y_{H,t}^{M*}(j)$ produced by a Foreign-located plant owned by a Home MNE is $p_{H,t}^{M*}(j) = \frac{P_{H,t}^{M*}(j)}{P_t^*}$. Both are expressed in terms of the Foreign final good. $Q_t = \frac{\epsilon_t P_t^*}{P_t}$ denotes the real exchange rate, with ϵ_t being the nominal exchange rate, defined as the Home currency price of the Foreign currency.

Intermediate firms are assumed to use Producer Currency Pricing (PCP), where prices are set in the currency of the producers.[4] As a result, the optimal price setting is

$$P_{H,t}(j) = \epsilon_t P_{H,t}^*(j) \tag{28}$$

$$\epsilon_t P_{F,t}^*(j) = P_{F,t}(j) \tag{29}$$

Equivalently in real terms, $p_{H,t}(j) = Q_t p_{H,t}^*(j)$ and $p_{F,t}^*(j) = \frac{p_{F,t}(j)}{Q_t}$. Prices are a constant markup over marginal costs. Under this specification, exchange rate pass-through is one-to-one.

Export and foreign direct investment dynamics are similar to Alessandria and Choi (2007), in which firms need to pay fixed costs $f^{\varsigma_{t-1}}(\varsigma_t)$ that are in terms of units of foreign labour. Firms that only sold to the Home market in the previous period, i.e. $j \in \mathcal{L}_{t-1}$, can choose to stay in the Home market or export to the Foreign market with export market entry costs $f^E(L)$. For firms that exported in the previous period, i.e. $j \in \mathcal{E}_{t-1}$, they can choose to exit the export market and only sell locally, or continue to export at a fixed export continuation cost $f^E(E)$, or operate

[4]See Corsetti and Pesenti (2009) for an overview of different pricing schedules.

a Foreign plant with MNE entry costs $f^M(E)$. Multinational firms in the previous period, i.e. $j \in \mathcal{M}_{t-1}$, can continue to operate as a MNE with continuation costs $f^M(M)$ or export with a "continuation" cost equivalent to the size of export continuation cost $f^E(M) = f^E(E)$. Entry costs are assumed to be higher than continuation costs, thus for export costs, $f^E(L) > f^E(E)$ and for MNE costs, $f^M(E) > f^M(M)$.

The transition relative frequencies between states are listed in Table 1. Monopolistically competitive firms have endogenous export and MNE entry and exit decisions every period. They are not allowed to jump directly between being a local firm and being a multinational firm for analytical tractability.

Table 1: **Transition Relative Frequencies**

	$j \in \mathcal{L}_t$	$j \in \mathcal{E}_t$	$j \in \mathcal{M}_t$	Σ
$j \in \mathcal{L}_{t-1}$	$\mathbb{P}_{LL,t}$	$\mathbb{P}_{LE,t}$	0	1
$j \in \mathcal{E}_{t-1}$	$\mathbb{P}_{EL,t}$	$\mathbb{P}_{EE,t}$	$\mathbb{P}_{EM,t}$	1
$j \in \mathcal{M}_{t-1}$	0	$\mathbb{P}_{ME,t}$	$\mathbb{P}_{MM,t}$	1

Notes: Monopolistically competitive firms have endogenous export and MNE entry and exit decisions every period. For analytical tractability, they are not allowed to jump directly between being a local firm and a multinational firm.

The law of motion for exporters is thus:

$$|\mathcal{E}_t| = (1 - |\mathcal{E}_{t-1}| - |\mathcal{M}_{t-1}|) \times \mathbb{P}_{LE,t} + |\mathcal{E}_{t-1}| \times \mathbb{P}_{EE,t} + |\mathcal{M}_{t-1}| \times \mathbb{P}_{ME,t}, \tag{30}$$

whereas the law of motion for multinational firms is:

$$|\mathcal{M}_t| = |\mathcal{E}_{t-1}| \times \mathbb{P}_{EM,t} + |\mathcal{M}_{t-1}| \times \mathbb{P}_{MM,t}. \tag{31}$$

Each intermediate firm has individual state variables $(n_{H,t}^{\varsigma-1}, z_t, \varsigma_{t-1})$ and aggregate state variables

(A_t, e^{ξ_t}). Hence, intermediate firms need to solve the following dynamic problem:[5]

$$V_t\left(n_{H,t}^{\varsigma_{-1}}, z_t, \varsigma_{t-1}; A_t, e^{\xi_t}\right) = \max_{\substack{\varsigma_t, s_{H,t}, n_{H,t+1},\\ l_{H,t}, l_{F,t}^M, k_{H,t}, k_{F,t}^M\\ p_{H,t}, p_{H,t}^*,\\ y_{H,t}, y_{H,t}^*}} \pi_t(j) + \mathbb{E}_t M_{t,t+1} V_{t+1}\left(n_{H,t+1}, z_{t+1}, \varsigma_t; A_{t+1}, e^{\xi_{t+1}}\right)$$
$$- \begin{cases} 0 & \text{if } \varsigma_t(j) = L \\ Q_t w_t^* f^E(\varsigma_{t-1}) & \text{if } \varsigma_t(j) = E \\ Q_t w_t^* f^M(\varsigma_{t-1}) & \text{if } \varsigma_t(j) = M \end{cases} \tag{32}$$

where firms maximize profit and pay the export cost $Q_t w_t^* f^E(\varsigma_{t-1})$ if they export and $Q_t w_t^* f^M(\varsigma_{t-1})$ if they operate as a multinational firm. $M_{t,t+1} = \beta \frac{MU_{C,t+1}}{MU_{C,t}}$ is the stochastic discount factor from the household problem, with $MU_{C,t}$ being the marginal utility of the household at time t.

Using (26) and (27), the optimality condition for the technological capital investment $s_{H,t}$ is

$$p_t^s = \mathbb{E}_t M_{t,t+1} V_{n,t+1}. \tag{33}$$

Given that the idiosyncratic productivity z_t shock is i.i.d. across firms, this means that the technological capital demand $n_{H,t+1}$ depends on the firm's status ς_t in the previous period but is independent of z_t and z_{t+1}.

For notational convenience, I now define the following partitions of the set of the possible firm statuses: $d \equiv \{L, E\}$ and $f = \{E, M\}$. I now discuss the thresholds between (1) being a local firm and exporting, and (2) exporting and being a multinational firm.

The first threshold exists for those with firm status $\varsigma_{t-1} \in d$ in the previous period. In particular, there is a productivity threshold $\bar{z}_t^d$ such that the value function for staying local V_t^L equals the value function for exporting V_t^E:[6]

$$V_t^L(n_{H,t}^d, \bar{z}_t^d, d) = V_t^E(n_{H,t}^d, \bar{z}_t^d, d). \tag{34}$$

The second threshold is for those with firm status $\varsigma_{t-1} \in f$ in the previous period. Hence, there is a productivity threshold $\hat{z}_t^f$ such that the value function for exporting V_t^E equals the value function

[5] For notational convenience, when the variable does not contain a superscript, it indicates all possible combinations of the states.

[6] With a slight abuse of notation, I denote d to be both the subset and the firm's actual status in the previous period.

for operating a plant in each country V_t^M:

$$V_t^E(n_{H,t}^f, \hat{z}_t^f, f) = V_t^M(n_{H,t}^f, \hat{z}_t^f, f). \tag{35}$$

By plugging in the labour and physical capital demands, as well as the optimal prices, into the profit function (27) within the value function (32), the threshold (34) becomes

$$\begin{aligned}
&Q_t w_t^* f^E(d) + p_t^s \left(n_{H,t+1}^E(j) - n_{H,t+1}^L(j)\right) \\
&\quad = \left[\frac{w_t}{\Xi}\right]^{1-\varepsilon\nu} \left(A_t z_{d,t}(j)\right)^{(\varepsilon-1)\nu} n_{H,t}^d(j)^{1-\nu} \left[\frac{\alpha}{1-\alpha}\frac{w_t}{r_t^k}\right]^{\alpha(\varepsilon\nu-1)} \\
&\qquad\quad \times \left[\left(\Gamma_{H,t} + \Gamma_{H,t}^{E*}\right)\left(\Gamma_{H,t} + e^{\xi_t}\Gamma_{H,t}^{E*}\right)^{\nu-1} - \Gamma_{H,t}{}^{\nu} - \frac{\Xi}{1-\alpha}\left[\left(\Gamma_{H,t} + e^{\xi_t}\Gamma_{H,t}^{E*}\right)^{\nu} - \Gamma_{H,t}{}^{\nu}\right]\right] \\
&\qquad + \mathbb{E}_t M_{t,t+1}\left[V_{t+1}(n_{H,t+1}^E(j), z_{t+1}(j), E) - V_{t+1}(n_{H,t+1}^L(j), z_{t+1}(j), L)\right],
\end{aligned} \tag{36}$$

where $\Xi = (1-\chi)(1-\alpha)\frac{\varepsilon-1}{\varepsilon}$ and $\nu = \frac{1}{1+(\varepsilon-1)\chi}$ for easier notation. $\Gamma_{H,t} = p_{H,t}{}^{\varepsilon} Y_{H,t}$ and $\Gamma_{H,t}^{E*} = \left(Q_t p_{H,t}^{E*}\right)^{\varepsilon} Y_{H,t}^{E*}$ denote the Home and Foreign market size that Home local firms and exporters have access to. The proof is in Appendix B.4.

The left hand side of the threshold condition is the extra cost paid by the intermediate firm if it chose to export rather than staying local, which includes the export entry cost $Q_t w_t^* f^E(d)$ that depends on the export status at the beginning of the period, as well as the cost of the higher technological capital $p_t^s(n_{H,t+1}^E - n_{H,t+1}^L)$ needed to serve the larger market. The right hand side of the threshold condition is the extra benefit from exporting, stemming from the larger market size, as well as the higher expected value in the next period due to the export continuation cost being lower than the export entry cost.

Similarly, the threshold between exporting and operating as a multinational firm in (35) can be rewritten in terms of the extra benefits and extra costs from being a multinational firm:

$$
\begin{aligned}
& Q_t w_t^* [f^M(f) - f^E(f)] + p_t^s \left(n_{H,t+1}^M(j) - n_{H,t+1}^E(j) \right) \\
& \quad = \left(A_t \hat{z}_t^f \right)^{(\varepsilon-1)\nu} {n_{H,t}^f}^{1-\nu} \left[\frac{\alpha}{1-\alpha} \right]^{\alpha(\varepsilon\nu-1)} \\
& \quad \times \Bigg\{ \left[\frac{w_t}{\Xi} \right]^{1-\varepsilon\nu} \left[\frac{w_t}{r_t^k} \right]^{\alpha(\varepsilon\nu-1)} \left[{\Gamma_{H,t}}^{\nu} - \left(\Gamma_{H,t} + \Gamma_{H,t}^{E*} \right) \left(\Gamma_{H,t} + e^{\xi_t} \Gamma_{H,t}^{E*} \right)^{\nu-1} - \frac{\Xi \left[{\Gamma_{H,t}}^{\nu} - (\Gamma_{H,t} + e^{\xi_t} \Gamma_{H,t}^{E*})^{\nu} \right]}{1-\alpha} \right] \\
& \qquad + \left[\frac{w_t^*}{\Xi} \right]^{1-\varepsilon\nu} \left[\frac{w_t^*}{r_t^{k*}} \right]^{\alpha(\varepsilon\nu-1)} Q_t \left[1 - \frac{\Xi}{1-\alpha} \right] {\Gamma_{H,t}^{M*}}^{\nu} \Bigg\} \\
& \quad + \mathbb{E}_t M_{t,t+1} \left[V_{t+1}(n_{H,t+1}^M(j), z_{t+1}(j), M) - V_{t+1}(n_{H,t+1}^E(j), z_{t+1}(j), E) \right],
\end{aligned}
\tag{37}
$$

where $\Gamma_{H,t}^{M*} = {p_{H,t}^{M*}}^{\varepsilon} Y_{H,t}^{M*}$ denotes the Foreign market size that a Home-owned multinational firm has access to.

4.4 Monetary Policy

The monetary authority sets the nominal interest rate R_t following an inertial Taylor rule to stabilize output growth and inflation. Hence monetary policy takes the following form:

$$
R_t = (R_{t-1})^{\phi_R} \left[R \left(\frac{\pi_t^P}{\pi^P} \right)^{\phi_\pi} \left(\frac{Y_t}{Y_{t-1}} \right)^{\phi_Y} \right]^{1-\phi_R} \tag{38}
$$

where $\phi_R \in (0,1)$ is the coefficient of interest rate smoothing, and R and π^P are the steady state nominal interest rate and inflation, respectively. $\phi_\pi > 0$ and $\phi_Y > 0$ govern the weights of inflation $\pi_t^P = \frac{P_t}{P_{t-1}}$ and output growth $\frac{Y_t}{Y_{t-1}}$ respectively.

4.5 Government

The government budget constraint is as follows:

$$
\frac{e^{\tau_t} P_{F,t}^E}{P_t} Y_{F,t}^E = T_t \tag{39}
$$

The government collects import tariffs to pay for transfers to households, maintaining a balanced budget in each period.

4.6 Equilibrium

A competitive equilibrium is a process of prices and quantities such that,

(i) Taking prices $\{w_t, r_t^k, p_t^s, \epsilon_t, R_t, R_t^*, P_t, T_t\}_{t=0}^{\infty}$ as given, the allocation $\{C_t, L_t, B_{H,t+1}, B_{F,t+1}, I_t, K_t, S_t\}_{t=0}^{\infty}$ solves the household's problem.

(ii) Taking prices $\{P_{H,t}, P_{F,t}^M, P_{F,t}^E, \tau_t\}_{t=0}^{\infty}$ as given, the allocation $\{P_t(i), Y_t(i)Y_{H,t}(i), Y_{F,t}^M(i), Y_{F,t}^E(i)\}_{t=0}^{\infty}$ solves the retailer's problem.

(iii) Taking prices $\{w_t, w_t^*, r_t^k, r_t^{k*}, p_t^s, Q_t, e^{\xi_t}, f^E(L), f^E(E), f^M(E), f^M(M)\}_{t=0}^{\infty}$ as given, the allocation $\{k_{H,t}, k_{F,t}(j), l_{H,t}(j), l_{F,t}(j), s_{H,t}(j), P_{H,t}(j), p_{H,t}^*(j), p_{HF,t}^*(j), y_{H,t}(j), y_{H,t}^*(j)\}_{t=0}^{\infty}$ solves the intermediate firm's problem.

(iv) The gross nominal interest rate $\{R_t\}_{t=0}^{\infty}$ satisfies the Taylor rule in Equation (38).

(v) The import tariffs $\{\tau_t, T_t\}_{t=0}^{\infty}$ satisfies the government budget constraint in Equation (39).

(vi) The labour market clears.

$$\begin{aligned} L_t = &(1 - |\mathcal{E}_t| - |\mathcal{M}_t|)\, l_{H,t}^L + |\mathcal{E}_t|\, l_{H,t}^E + |\mathcal{M}_t|\, l_{H,t}^M + |\mathcal{M}_t^*|\, l_{H,t}^{M*} \\ &+ (1 - |\mathcal{E}_{t-1}^*| - |\mathcal{M}_{t-1}^*|)\, \mathbb{P}_{LE,t}^* f^E(L) + |\mathcal{E}_{t-1}^*|\, \mathbb{P}_{EE,t}^*\, f^E(E) \\ &+ |\mathcal{E}_{t-1}^*|\, \mathbb{P}_{EM,t}^*\, f^M(E) + |\mathcal{M}_{t-1}^*|\, \mathbb{P}_{MM,t}^*\, f^M(M), \end{aligned}$$

where (a) $l_{H,t}^L \equiv \int_{j\in\mathcal{L}_t} l_{H,t}(j)\mathrm{d}j$, (b) $l_{H,t}^E \equiv \int_{j\in\mathcal{E}_t} l_{H,t}(j)\mathrm{d}j$, (c) $l_{H,t}^M \equiv \int_{j\in\mathcal{M}_t} l_{H,t}(j)\mathrm{d}j$, and (d) $l_{H,t}^{M*} = \int_{j\in\mathcal{M}_t^*} l_{H,t}^*(j)\mathrm{d}j$.

(vii) The technological capital market clears.

$$N_{t+1} = (1 - |\mathcal{E}_t| - |\mathcal{M}_t|)\, n_{H,t+1}^L + |\mathcal{E}_t|\, n_{H,t+1}^E + |\mathcal{M}_t|\, n_{H,t+1}^M,$$

where (a) $n_{H,t+1}^L \equiv \int_{j\in\mathcal{L}_t} n_{H,t+1}(j)\mathrm{d}j$, (b) $n_{H,t+1}^E \equiv \int_{j\in\mathcal{E}_t} n_{H,t+1}(j)\mathrm{d}j$, and (c) $n_{H,t+1}^M \equiv \int_{j\in\mathcal{M}_t} n_{H,t+1}(j)\mathrm{d}j$

(viii) The physical capital market clears.

$$K_t = (1 - |\mathcal{E}_t| - |\mathcal{M}_t|)\, k_{H,t}^L + |\mathcal{E}_t|\, k_{H,t}^E + |\mathcal{M}_t|\, k_{H,t}^M + |\mathcal{M}_t^*|\, k_{H,t}^{M*},$$

where (a) $k^L_{H,t} \equiv \int_{j\in\mathcal{L}_t} k_{H,t}(j)\mathrm{d}j$, (b) $k^E_{H,t} \equiv \int_{j\in\mathcal{E}_t} k_{H,t}(j)\mathrm{d}j$, (c) $k^M_{H,t} \equiv \int_{j\in\mathcal{M}_t} k_{H,t}(j)\mathrm{d}j$, and (d) $k^{M*}_{H,t} = \int_{j\in\mathcal{M}^*_t} k^*_{H,t}(j)\mathrm{d}j$.

(ix) The final good market clears, satisfying Equation (7).

(x) The bundle market clears.

$$\int_{i\in[0,1]} Y_{H,t}(i)\,\mathrm{d}i = Y_{H,t}$$

$$\int_{i\in[0,1]} Y^M_{H,t}(i)\,\mathrm{d}i = Y^M_{H,t}$$

$$\int_{i\in[0,1]} Y^E_{F,t}(i)\,\mathrm{d}i = Y^E_{F,t}$$

(xi) The intermediate good market clears, satisfying Equations (10), (12), (11).

(xii) Bond markets clear. $\frac{B_{H,t}}{P_t} + \frac{B^*_{H,t}}{\epsilon_t P^*_t} = 0$

where analogous variables and equations exist for Foreign.

5 Solving the Model

5.1 Solution Method

The model is solved using the third-order perturbation method via Dynare 4.4.3 on Matlab 2019a. Caldara et al. (2012) solved a DSGE model with recursive preferences and stochastic volatility using several computational methods and showed that second- and third-order perturbation methods are accurate and computationally efficient compared to Chebyshev polynomials and value function iteration. Fernández-Villaverde et al. (2011) showed that shocks to volatility will not play a role in the first-order approximation of the policy functions as the approximation is certainty equivalent. In the second-order approximation, volatility will only appear in the cross-product terms with the level innovations. Hence, third-order perturbation is necessary here to isolate the effects of trade policy uncertainty shocks. This method has been used in the literature, such as Fernández-Villaverde et al. (2011), Basu and Bundick (2017), and Bonciani and Oh (2019), to show the direct effects of uncertainty.

Impulse response functions are calculated surrounding the risky (or stochastic) steady state where risk-averse agents expect future risk but realization of shocks is zero. This differs from the deterministic steady state from the perfect foresight case.[7]

5.2 Calibration

The export and multinational firms entry and continuation costs are calibrated to match 2017 U.S. export and multinational firms. The masses of exporters and multinational firms are calculated using the U.S. Census and the Bureau of Economic Analysis data on the number of firms.

Table 2: **Model Parameters**

Parameter	Description	Target
$\mathbb{P}_{EL}$	Export Market Exit Rate	0.0099
$\mathbb{P}_{ME}$	MNE Exit Rate	0.0033
$\lvert\mathcal{E}\rvert$	Mass of Exporters	$\frac{72{,}230}{291{,}263} = 0.24799$
$\lvert\mathcal{M}\rvert$	Mass of Multinational Firms	$\frac{1{,}310}{291{,}263} = 0.00450$

Notes: The per-period export and MNE entry and continuation costs in the model are calibrated to target the masses of exporters and multinational firms, as well as the steady state exit rates of exporters and multinational firms. Masses of exporters and multinational firms are calculated using the 2017 U.S. Census and the Bureau of Economics Analysis data while the exit rates are inferred from Caldara et al. (2020) and Gumpert et al. (2020).

Export market exit rates are found from Caldara et al. (2020), while multinational operation exit rates are estimated from Gumpert et al. (2020), which stated that exit rates of exporters are higher than those of affiliates of MNEs, reaching two- to threefold compared to new affiliates.

Using the targets in Table 2 and the law of motion for multinational firms (31) in steady state form $|\mathcal{M}| = |\mathcal{E}| \times \frac{\mathbb{P}_{EM}}{\mathbb{P}_{ME}}$, I can pin down $\mathbb{P}_{EM}$. Similarly, the targets and the law of motion for exporters (30) in steady state $|\mathcal{E}| = \frac{\mathbb{P}_{LE}}{\mathbb{P}_{LE}+\mathbb{P}_{EL}+\mathbb{P}_{EM}\times\mathbb{P}_{LE}/\mathbb{P}_{ME}}$ can help pin down $\mathbb{P}_{LE}$, aong with $\mathbb{P}_{LL} = 1 - \mathbb{P}_{LE}$. Finally, $P_{EE} = 1 - \mathbb{P}_{EL} - \mathbb{P}_{EM}$. These are then translated into quarterly frequency.

The rest of the parameters are mostly taken from the literature and are listed in Table 3. Parameters for the utility function come from standard open-economy models such as Backus et al. (1994), with the discount factor β set to 0.99 and the coefficient of relative risk aversion σ set to

[7]While Coeurdacier et al. (2011) proposed to solve for the risky steady state by using a second-order approximation of the Euler equation, it was proved to be inaccurate method (Den Haan et al., 2015).

2. Other parameters for the GHH utility come from Caldara et al. (2020), with the disutility of labour ψ set to 29 and the inverse Frisch elasticity μ set to 1. The habit formation parameter b follows Boldrin et al. (2001) and is chosen to be 0.73.

The elasticity of substitution between the differentiated retail goods ϕ follows from Caldara et al. (2020) and is set to 11. The elasticity between intermediate varieties ε follows from Alessandria et al. (2014), thus it is set to be 5 to generate a producer markup $\frac{\varepsilon}{\varepsilon-1}$ of 25 percent. The home-owned firm bias ω follows Backus et al. (1994) and is set to be 0.85. I allow domestic and foreign goods to be complements, and hence set the elasticity of substitution to be $\theta < 1$. On the other hands, I assume households treat foreign exported goods and MNE goods as substitutes, and hence the elasticity of substitution is assumed to be $\eta > 1$.

The investment adjustment cost κ is set to 10 à la Caldara et al. (2020) to ensure that the unconditional standard deviation of capital investment is about twice as large as that of GDP. Depreciation rates of technological capital and physical capital are taken from Kapička (2012), where technological capital is expected to depreciate at a higher rate as it includes R&D investment, which has a higher depreciation rate according to Bureau of Labor Statistics data (see Kung (2015) and Bonciani and Oh (2019)). For the firm side, the technological capital share χ and the physical capital share $\alpha(1-\chi)$ follow from Kapička (2012).

Parameters for the intermediate firms' productivity follow from Caldara et al. (2020). The AR(1) aggregate productivity A_t has a persistence ρ of 0.95, and the firm specific productivity $z_t(j)$ has a standard deviation of 0.5. The Rotemberg price adjustment parameter ρ_P is set to be 561.15, which to a first order approximation implies a Calvo parameter of 0.975 (Lombardo and Vestin, 2008), hence firms on average update price every 40 quarters.

The inertial monetary rule has inertia parameter ϕ_R of 0.85, which is consistent with the "modernized" Taylor rule set by the Fed (Kliesen, 2019). The weight on inflation in the Taylor rule ϕ_π is set to be 1.25, whereas the weight on output ϕ_Y is set to be 0.35.

The persistence of the trade costs ρ_τ is set to 0.99, while the persistence of import tariff volatility ρ_{σ_τ} is set to 0.96 following Caldara et al. (2020), who estimated the import tariff AR(1) with AR(1) process using 1960:Q1 to 2018:Q4 U.S. aggregate import tariff data.[8]

[8]The tariffs series is reconstructed using product group-level data and presented in Appendix A.

Table 3: **Model Parameters**

Parameter	**Description**	**Value**
	Households	
β	Discount factor	0.99
b	Habit formation	0.73
ψ	Disutility of Labour	29
σ	Risk Aversion	2
μ	Inverse Frisch Elasticity	1
	Firms	
ϕ	Elasticity of substitution between retail goods	11
ω	Relative share of Home bundles	0.85
θ	Elasticity of substitution between Home and Foreign bundles	0.5
ν	Relative share of Foreign-exported bundle	0.5
η	Elasticity of substitution between exported & MNE bundles	1.5
ρ_P	Price adjustment parameter	561.15
ε	Elasticity of substitution between varieties	5
χ	Technological capital share	0.1041
α	Physical capital share	0.3728
δ_n	Rate of technological capital depreciation	0.04
δ_k	Rate of physical capital depreciation	0.01125
κ	Capital investment adjustment cost	10
ρ	Persistence of aggregate productivity	0.95
σ_z	Standard deviation of idiosyncratic shock	0.5
	Entry and Exit	
$f^E(L)$	Export entry cost	0.8204
$f^E(E)$	Export continuation cost	0.0465
$f^M(E)$	MNE entry cost	25.3584
$f^M(M)$	MNE continuation cost	0.6567
	Monetary Policy Taylor Rule	
ϕ_R	Inertia Parameter	0.85
ϕ_π	Coefficient on inflation	1.25
ϕ_Y	Coefficient on output growth	0.35
	Uncertainty Processes	
ρ_ξ	Persistence of non-tariff measures	0.99
ρ_{σ_ξ}	Persistence of non-tariff measures volatility	0.96

6 Model Experiments

This section studies the effect of trade policy uncertainty in the form of non-tariff measures uncertainty. As the model extends from Caldara et al. (2020) by allowing firms to be multinational firms, I first shut down this channel to recap how trade policy uncertainty would affect exporters.

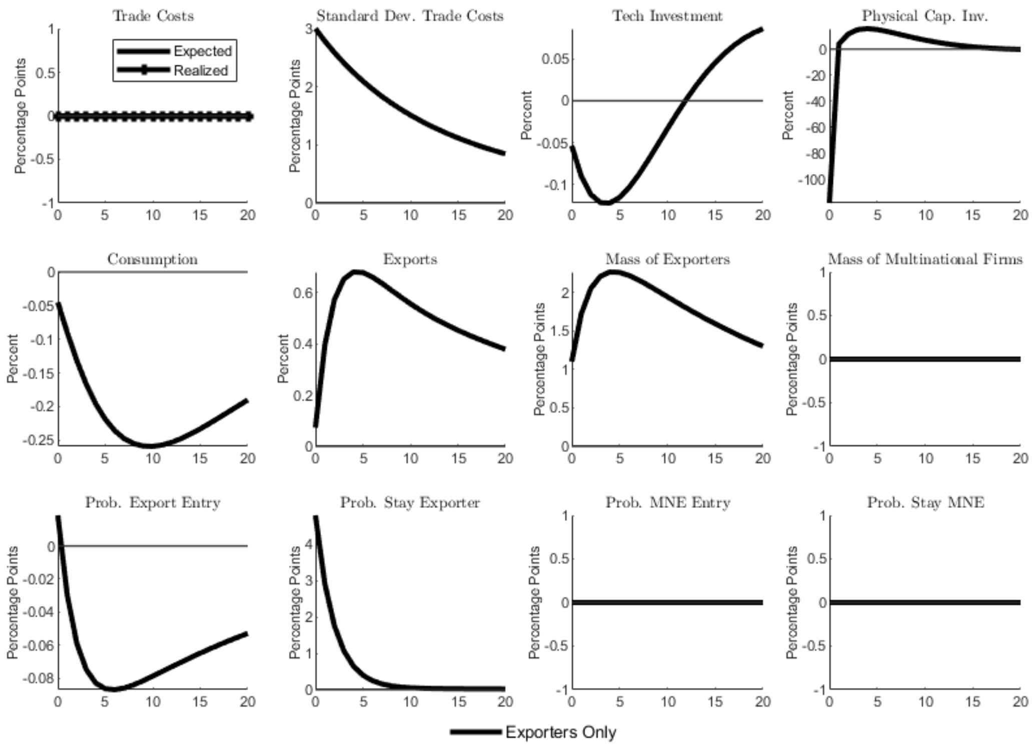
6.1 Without Multinational Firms

To return to an exporters-only framework, the steady state probability $\mathbb{P}_{EM}$ of switching from being an exporter to being a multinational firm is set to 0 and the steady state probability of the reverse $\mathbb{P}_{ME}$ is set to 1. I now study the effects of uncertain trade costs in this framework. The impulse response functions are shown in Figure 5.

This case is similar to the one studied in Caldara et al. (2020), with the main difference being that Caldara et al. (2020) had uncertainty around tariffs while the case presented here has uncertainty modeled as an iceberg trade cost.[9] In either case, the basic intuition is that when exporters face uncertainty over trade costs, they would rather stay on as an exporter than leave. This is because if they leave the export market, they would need to incur a high export entry cost if they were to reenter. Hence, they would rather pay the smaller export continuation cost. Since the decrease in the probability export entry is relatively smaller than the probability of existing exporters staying as exporters, overall there is an increase in the mass of exporters. Due to nominal rigidities, retailers charge a higher markup, thus consumption and investment both decrease due to the usual precautionary saving channel. As a result, GDP falls under the trade policy uncertainty shock.

[9] For a direct comparison with Caldara et al. (2020), the coefficient on output in the Taylor rule ϕ_Y needs to be set to 0. There is no technological capital in the Caldara et al. (2020) model and firms instead accumulate physical capital.

Figure 5: **IRFs of Trade Policy Uncertainty Effects without Multinational Firms**

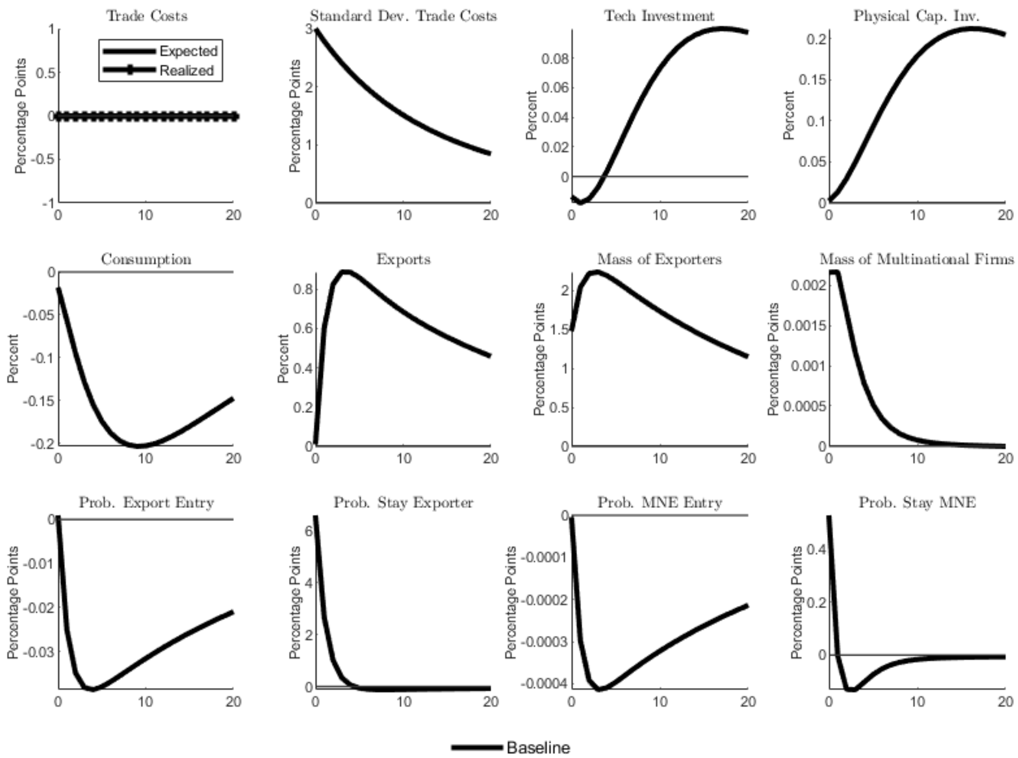


Notes: By forcing the steady state probability $\mathbb{P}_{EM}$ of switching from being an exporter to being a multinational firm to 0 and the steady state probability of the reverse $\mathbb{P}_{ME}$ to 1, the model now only allows exporters and shuts down the multinational firms channel. The figure shows impulse response functions to uncertain non-tariff measures, modeled as iceberg trade costs.

6.2 With Multinational Firms

Taking away the restrictions set in the previous section, the full model now allows for the existence of multinational firms. Figure 6 shows the impulse response functions to uncertain trade costs. As seen in the figure, the economy reacts differently to trade policy uncertainty shocks now that firms can serve a foreign market by being a multinational firm. Consumption still decreases, as the precautionary saving channel is still present, but both technological and physical capital investment now increase. Hence, there is now a lack of comovement between consumption and investment. While New Keynesian models tend to have a comovement between consumption and investment (see Basu and Bundick (2017)), the lackthereof has also been documented, such as in Di Pace and Görtz (2021). As suggested in Di Pace and Görtz (2021), the lack of comovement can be caused by the absence of frictions in credit provisions.

Figure 6: **IRFs of Trade Policy Uncertainty Effects with Multinational Firms**



Notes: The figure shows the impulse response functions to uncertain trade costs in a model with exporters and multinational firms.

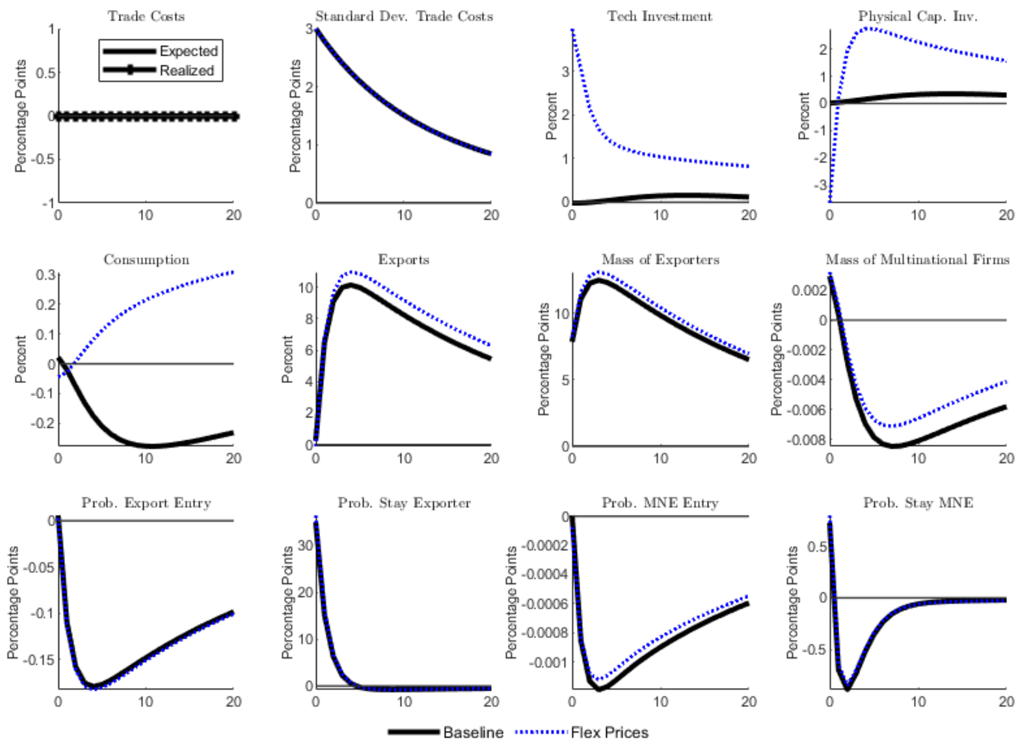
From the figure, it can be seen that the probabilities of staying as an exporter and staying as a MNE both increase immediately upon a trade policy uncertainty shock. The two increases are due to export (or MNE) entry costs being higher than export (or MNE) continuation costs, thus firms would rather stay as an exporter (or MNE) than leave. The increase in the probability of staying as an exporter is (1) greater than the increase in the probability of staying as a MNE, and (2) greater than the probability in the exporters-only case. The former is because exporter continuation cost is lower than the MNE continuation cost. Hence, it is easier for firms to reenter as exporter than to reenter as a MNE. The latter is because if exporters leave the market, they would need to reenter to be an exporter before having the opportunity to become a multinational firm, thus incurring two high entry costs. This makes it more likely for firms to stay as exporters. On the other hand, the probabilities of export and MNE entries only decrease by a small amount, as the uncertainty over trade costs is resolved quickly. The combination of these four changes thus drive both the mass of exporters and the mass of multinational firms up following uncertainty shocks to trade policies. The change in the mass of exporters remains positive

6.3 Robustness Checks: Effects of Nominal Rigidities

To understand the results better, I now conduct a sensitivity analysis with flexible prices (Flex Prices). Figure 7 plots the impulse response functions from the baseline case, i.e. in a model with MNEs, with the results from the alternative experiment. It can be seen that the predictions on the entry, exit and mass of exporters and multinational firms are highly similar for these two cases.

Under "Flexible Prices", the price adjustment parameter ρ_P is set to 0 and thus retailers will only charge a constant markup over the marginal cost. As seen in the figure, nominal rigidities are key for the impact on consumption and investment. This is because when there is trade cost uncertainty, the cost of the imported bundle $Y^E_{F,t}$, and thus the marginal cost of producing the retail good, is unknown. As a result, when price adjustment is costly, retailers would charge a higher markup to avoid selling at too low of a price in the future. The higher markup thus would lead to a fall in consumption. Absent the higher markup, consumption and investment expand via standard Oi-Hartman-Abel effects. However, it is worth noting that the predictions on the movements of exporters and MNEs remain largely the same as the base case.

Figure 7: **Robustness Checks for Trade Policy Uncertainty Effects**



Notes: Impulse response functions from the baseline case with MNEs are compared with the case where price adjustment is set to 0 (Flexible Prices).

7 Discussions and Conclusion

This paper serves as a first attempt to explore the effects of trade policy uncertainty in a model with multinational firms. It uses a DSGE model that allows for endogenous export and MNE entry to show that under non-tariffs barrier uncertainty, the mass of exporters will increase as exporters would continue to stay as exporter to avoid paying the entry cost if they exited the market, and to accumulate export experience to become a multinational firm. The mass of multinational firms also increases, though the increment is much more short-lived. As exporters and multinational firms are known to be more productive than local firms, this implies that the long run productivity can be affected when there is high trade policy uncertainty in the economy. This would be an even greater impact if there is technological spillover between countries (see Bianchi et al. (2019) and Bonciani and Oh (2019) for a discussion on endogenous growth due to R&D technology spillovers).

There is much left to explore in relation to this paper. I assumed that firms enter multinational operation via greenfield entry, thus not capturing another mode of multinational operation entry – cross-border merger and acquisition. Antràs and Yeaple (2014) showed that while establishing a new facility in a foreign country is not fundamentally different from the acquisition of an existing foreign firm, they are not perfect substitutes. Moreover, foreign direct investment in this paper was also restricted to horizontal FDI, where firms expand by replicating their production overseas to save on trade costs. Alternative models can explore vertical FDI, where firms move some parts of the production process overseas due to cost differences. Finally, given that this is a two-country model, it is unable to account for third-country sales by MNE affiliates, where sales go neither to the host nor the source country, but to another country altogether. In a world where global value chains are a prominent feature, it is likely that trade policy uncertainty can spill over, as MNEs seek to set plants in countries that have many secure bilateral trade deals in place, and are geographically close to their desired markets.

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A Data

The targetted mass of exporters and mass of multinational affiliates in Table 2 are calculated using 2017 data from the U.S. the Census and the Bureau of Economic Analysis. In particular, the total number of firms is taken from Census and the number of multinational firms is taken from Table I.A 2 "Selected Data for Foreign Affiliates and U.S. Parents in All Industries".

Given that the import tariffs used in generating Caldara et al. (2020)'s import tariff uncertainty proxy are measured at the aggregate level, there is concern that the series would be affected by the composition of the imports. As any misspecification for tariff level would be absorbed in the residual, σ_{τ_t} (or might be inaccurate in capturing import tariffs uncertainty. As a result, the tariffs series is reconstructed using product group-level data.

Import data are collected from the USA Trade Online, which is provided by the U.S. Census Bureau. In particular, monthly data from 2002 January to 2018 December on the import value of goods that include cost, insurance, and freight (CIF value) are chosen. Tariffs data are sourced from the World Integrated Trade Solution. The effectively applied (AHS) average (%) tariff is then weighted by the corresponding trade value of the products in each product group at the HS-2 digit level.

After re-estimating the new series using particle filter with 10,000 burnins and 60,000 draws (see Fernández-Villaverde et al. (2011) for details in estimating the parameters of an AR(1) with AR(1) process), the persistence of the tariffs ρ_τ is estimated to be 0.99, while the persistence of the import tariff volatility ρ_{σ_τ} is estimated to be 0.76.

B Proofs

B.1 Intermediate Firm's Optimal Price Setting

The cost minimization problem of an intermediate firm that only serves the local market $j \in \mathcal{L}_t$ is as follows

$$\min_{l_t(j)} \; w_t l_t(j) + r_t^k k_t(j) + p_t^s s_t(j)$$

$$\text{s.t. } A_t z_t(j) n_{H,t}^{\varsigma-1}(j)^{\chi} \left(k_t(j)^{\alpha} l_t(j)^{1-\alpha}\right)^{1-\chi} = y_{H,t}(j)$$

Hence Lagrangian is:

$$\mathcal{L} = w_t l_t(j) + p_t^s s_t(j) + \Phi_t(j) \left[A_t z_t(j) n_{H,t}^{\varsigma-1}(j)^{\chi} \left(k_{H,t}(j)^{\alpha} l_{H,t}(j)^{1-\alpha}\right)^{1-\chi} - y_{H,t}(j)\right]$$

where $\Phi_t(j)$ can be interpreted as the marginal cost.

First-order condition of the Lagrangian is:

$$w_t = \Phi_t(j)\,(1-\chi)(1-\alpha) A_t z_t(j) n_{H,t}^{\varsigma-1}(j)^{\chi} \left(k_{H,t}(j)^{\alpha} l_{H,t}(j)^{-\alpha}\right)^{1-\chi}$$

And thus, the marginal cost is:

$$\Phi_t(j) = \frac{w_t l_t(j)}{(1-\chi)(1-\alpha) A_t z_t(j) n_{H,t}^{\varsigma-1}(j)^{\chi} \left(k_{H,t}(j)^{\alpha} l_{H,t}(j)^{1-\alpha}\right)^{1-\chi}} \tag{A.1}$$

The firm maximizes profit by setting its own optimal price:

$$\max_{p_{H,t}(j)} \; \left[p_{H,t}(j) - \Phi_t(j)\right] y_{H,t}(j)$$

Substituting in the intermediate variety demand function (18) for $y_{H,t}(j)$, the maximization problem becomes:

$$\max_{p_{H,t}(j)} \; \left[p_{H,t}(j) - \Phi_t(j)\right] \left[\frac{p_{H,t}(j)}{p_{H,t}}\right]^{-\varepsilon} Y_{H,t}$$

First-order condition is:

$$p_{H,t}(j) = \frac{\varepsilon}{\varepsilon - 1} \Phi_t(j)$$

which means that optimal price setting requires charging a constant marginal markup over marginal costs. As a result, the intermediate variety price set by a Home local firm $j \in \mathcal{L}_t$ or by a Home exporter $j \in \mathcal{E}_t$ or by a Home plant owned by a Home multinational firm $j \in \mathcal{M}_t$ is

$$p_{H,t}(j) = \frac{\varepsilon}{\varepsilon - 1} \frac{w_t l_{H,t}(j)}{(1-\chi)(1-\alpha) A_t z_t(j) n_{H,t}^{\varsigma-1}(j)^{\chi} \left(k_{H,t}(j)^{\alpha} l_{H,t}(j)^{1-\alpha}\right)^{1-\chi}}, \quad j \in \{\mathcal{L}_t, \mathcal{E}_t, \mathcal{M}_t\} \quad \text{(A.2)}$$

Similarly, the price of an intermediate variety sold to the Foreign country set by a Home exporter is

$$p_{H,t}^{E*}(j) = \frac{1}{Q_t} \frac{\varepsilon}{\varepsilon - 1} \frac{w_t l_{H,t}^{E}(j)}{(1-\chi)(1-\alpha) A_t z_t(j) n_{H,t}^{\varsigma-1}(j)^{1-\chi} \left(k_{H,t}^{E}(j)^{\alpha} l_{H,t}^{E}(j)^{1-\alpha}\right)^{1-\chi}}, \quad j \in \mathcal{E}_t \quad \text{(A.3)}$$

while the price of an intermediate variety produced by a plant located in Foreign owned by a Home multinational firm is

$$p_{H,t}^{M*}(j) = \frac{1}{Q_t} \frac{\varepsilon}{\varepsilon - 1} \frac{Q_t w_t^{*} l_{F,t}^{M}(j)}{(1-\chi)(1-\alpha) A_t z_t(j) n_{H,t}^{\varsigma-1}(j)^{\chi} \left(k_{F,t}^{M}(j)^{\alpha} l_{F,t}^{M}(j)^{1-\alpha}\right)^{1-\chi}}, \quad j \in \mathcal{M}_t \quad \text{(A.4)}$$

B.2 Intermediate Firm's Input Demand

B.2.1 Physical Capital Demand

The first order condition for the physical capital demand of an intermediate firm is

$$r_t^k = \Phi_t \, (1-\chi) \, \alpha \, A_t z_t(j) n_{H,t}^{\varsigma-1}(j)^{\chi} (k_{H,t}(j)^{\alpha} l_{H,t}(j)^{1-\alpha})^{-\chi} \, k_{H,t}(j)^{\alpha-1} l_{H,t}(j)^{1-\alpha}$$

To simplify matters, substitute in the marginal cost Φ_t equation (A.1) found earlier to express the physical capital demand in terms of labor demand:

$$k_{H,t}(j) = \frac{\alpha}{1-\alpha} \frac{w_t}{r_t^k} l_{H,t}(j), \quad j \in \{\mathcal{L}_t, \mathcal{E}_t, \mathcal{M}_t\} \quad \text{(A.5)}$$

whilst the physical capital demand for the Foreign plant owned by a Home multinational firm is

$$k_{F,t}^{M}(j) = \frac{\alpha}{1-\alpha} \frac{w_t^{*}}{r_t^{k*}} l_{F,t}^{M}(j), \quad j \in \mathcal{M}_t \quad \text{(A.6)}$$

B.2.2 Technological Capital Demand

The first order condition for the technological capital demand of an intermediate firm is

$$p^s_t = \mathbb{E}_t\left[p^s_{t+1}(1-\delta_n) + \Phi_{t+1}\,\chi\, A_{t+1} z_{t+1}(j) n_{H,t+1}(j)^{\chi-1}(k_{H,t+1}(j)^{\alpha} l_{H,t+1}(j)^{1-\alpha})^{1-\chi}\right]$$

where the marginal cost Φ_{t+1} can be replaced by (A.1) set at time $t+1$.

The technological capital demands for a Home local firm, a Home exporter, and a Home multinational firm are thus:

$$p^s_t = \mathbb{E}_t M_{t,t+1}\left[(1-\delta_n)p^s_{t+1} + \frac{\chi}{(1-\chi)(1-\alpha)}\frac{w_{t+1} l_{H,t+1}(j)}{n_{H,t+1}(j)}\right], \quad j \in \{\mathcal{L}_t, \mathcal{E}_t\} \tag{A.7}$$

$$p^s_t = \mathbb{E}_t M_{t,t+1}\left[(1-\delta_n)p^s_{t+1} + \frac{\chi}{(1-\chi)(1-\alpha)}\left(\frac{w_{t+1} l_{H,t+1}(j)}{n_{H,t+1}(j)} + \frac{w^*_{t+1} Q_{t+1} l_{F,t+1}(j)}{n_{H,t+1}(j)}\right)\right], \quad j \in \mathcal{M}_t \tag{A.8}$$

B.2.3 Labour Demand

To solve for the labour demands of the local firms and exporters, first substitute in the demands for the intermediate varieties (18) and the Home-equivalent of (19) in the resource constraint / production function:

$$y_{H,t}(j) + E_t(j)\, e^{\xi_t}\, y^*_{H,t}(j) = A_t z_t(j) n^{\varsigma-1}_{H,t}(j)^{\chi}\left(k_{H,t}(j)^{\alpha} l_{H,t}(j)^{1-\alpha}\right)^{1-\chi}$$

$$\left[\frac{p_{H,t}(j)}{p_{H,t}}\right]^{-\varepsilon} Y_{H,t} + E_t(j)\, e^{\xi_t}\left[\frac{p^*_{H,t}(j)}{p^{E*}_{H,t}}\right]^{-\varepsilon} Y^{E*}_{H,t} = A_t z_t(j) n^{\varsigma-1}_{H,t}(j)^{\chi}\left(k_{H,t}(j)^{\alpha} l_{H,t}(j)^{1-\alpha}\right)^{1-\chi}$$

$$\left[p_{H,t}(j)\right]^{-\varepsilon}\left[p_{H,t}{}^{\varepsilon} Y_{H,t} + E_t(j)\, e^{\xi_t}\left(Q_t p^{E*}_{H,t}\right)^{\varepsilon} Y^{E*}_{H,t}\right] = A_t z_t(j) n^{\varsigma-1}_{H,t}(j)^{\chi}\left(k_{H,t}(j)^{\alpha} l_{H,t}(j)^{1-\alpha}\right)^{1-\chi}$$

Substitute in the corresponding intermediate variety price (A.2) and let $\Xi = (1-\chi)(1-\alpha)\,\frac{\varepsilon-1}{\varepsilon}$

$$\left[\frac{w_t}{\Xi}\frac{l_{H,t}(j)}{A_t z_t(j) n^{\varsigma-1}_{H,t}(j)^{\chi}\left(k_{H,t}(j)^{\alpha} l_{H,t}(j)^{1-\alpha}\right)^{1-\chi}}\right]^{-\varepsilon}\left[p_{H,t}{}^{\varepsilon} Y_{H,t} + E_t(j)\, e^{\xi_t}\left(Q_t p^{E*}_{H,t}\right)^{\varepsilon} Y^{E*}_{H,t}\right]$$
$$= A_t z_t(j) n^{\varsigma-1}_{H,t}(j)^{\chi}\left(k_{H,t}(j)^{\alpha} l_{H,t}(j)^{1-\alpha}\right)^{1-\chi}$$

Substitute in the physical capital demand (A.5),

$$\left[\frac{w_t}{\Xi}\right]^{-\varepsilon} l_{H,t}(j)^{(1-\varepsilon)\chi-1}\left[p_{H,t}{}^{\varepsilon} Y_{H,t} + \varsigma_t(j)\, e^{\xi_t}\left(Q_t p_{H,t}^{E*}\right)^{\varepsilon} Y_{H,t}^{E*}\right] = \left(A_t z_t(j)\right)^{1-\varepsilon} n_{H,t}^{\varsigma-1}(j)^{\chi(1-\varepsilon)}\left[\frac{\alpha}{1-\alpha}\frac{w_t}{r_t^k}\right]^{\alpha(1-\chi)(1-\varepsilon)}$$

By letting $\nu = \frac{1}{1+(\varepsilon-1)\chi}$, the labour demand of a local Home firm $j \in \mathcal{L}_t$ thus is

$$l_{H,t}^{L}(j) = \left[\frac{w_t}{\Xi}\right]^{-\varepsilon\nu}\left(A_t z_t(j)\right)^{(\varepsilon-1)\nu} n_{H,t}^{\varsigma-1}(j)^{1-\nu}\left[\frac{\alpha}{1-\alpha}\frac{w_t}{r_t^k}\right]^{\alpha(\varepsilon\nu-1)}\left(p_{H,t}{}^{\varepsilon} Y_{H,t}\right)^{\nu} \tag{A.9}$$

Similarly, the labour demand of a Home exporter $j \in \mathcal{E}_t$ is

$$l_{H,t}^{E}(j) = \left[\frac{w_t}{\Xi}\right]^{-\varepsilon\nu}\left(A_t z_t(j)\right)^{(\varepsilon-1)\nu} n_{H,t}^{\varsigma-1}(j)^{1-\nu}\left[\frac{\alpha}{1-\alpha}\frac{w_t}{r_t^k}\right]^{\alpha(\varepsilon\nu-1)}\left[p_{H,t}{}^{\varepsilon} Y_{H,t} + e^{\xi_t}\left(Q_t p_{H,t}^{E*}\right)^{\varepsilon} Y_{H,t}^{E*}\right]^{\nu} \tag{A.10}$$

The labour demand of a Home plant owned by a Home multinational firm $j \in \mathcal{M}_t$ is

$$l_{H,t}^{M}(j) = \left[\frac{w_t}{\Xi}\right]^{-\varepsilon\nu}\left(A_t z_t(j)\right)^{(\varepsilon-1)\nu} n_{H,t}^{\varsigma-1}(j)^{1-\nu}\left[\frac{\alpha}{1-\alpha}\frac{w_t}{r_t^k}\right]^{\alpha(\varepsilon\nu-1)}\left(p_{H,t}{}^{\varepsilon} Y_{H,t}\right)^{\nu} \tag{A.11}$$

while the labour demand of a Foreign plant owned by a Home multinational firm $j \in \mathcal{M}_t$ is

$$l_{F,t}^{M}(j) = \left[\frac{w_t^*}{\Xi}\right]^{-\varepsilon\nu}\left(A_t z_t(j)\right)^{(\varepsilon-1)\nu} n_{H,t}^{\varsigma-1}(j)^{1-\nu}\left[\frac{\alpha}{1-\alpha}\frac{w_t^*}{r_t^{k*}}\right]^{\alpha(\varepsilon\nu-1)}\left(p_{H,t}^{M*\varepsilon} Y_{H,t}^{M*}\right)^{\nu} \tag{A.12}$$

B.3 Bundle Price Index

The price index for the Home bundle sold to Home retailers $p_{H,t}$ is compromised of the intermediate variety price set by local Home firms, by Home exporter firms, and by Home-owned Home plants.

As a result, it can be expressed as:

$$p_{H,t} = \left[\int_{j\in[0,1]} p_{H,t}(j)^{1-\varepsilon}\mathrm{d}j\right]^{\frac{1}{1-\varepsilon}} \tag{A.13}$$

Substituting in the intermediate variety price (A.2)

$$p_{H,t} = \left[\int_{j\in[0,1]} \left(\frac{w_t}{\Xi}\right)^{1-\varepsilon} (A_t z_t(j))^{\varepsilon-1}\, n_{H,t}^{\varsigma-1}(j)^{\chi(\varepsilon-1)} k_{H,t}(j)^{\alpha(1-\chi)(\varepsilon-1)} l_{H,t}(j)^{(\chi+\alpha-\chi\alpha)(1-\varepsilon)} \mathrm{d}j\right]^{\frac{1}{1-\varepsilon}}$$

By substituting in the physical capital demand (A.5) for $k_{H,t}(j)$ and the labour demand (A.9) for $l_{H,t}(j)$, this can be expressed as

$$\begin{aligned} P_{H,t}{}^{1-\varepsilon} = &\int_{j\in\{\mathcal{L}_t,\mathcal{M}_t\}} \left[\frac{w_t}{\Xi}\right]^{1-\varepsilon\nu} (A_t z_t(j))^{\nu(\varepsilon-1)}\, n_{H,t}^{\varsigma-1}(j)^{1-\nu} \left[\frac{\alpha}{1-\alpha}\frac{w_t}{r_t^k}\right]^{\alpha(\varepsilon\nu-1)} \Gamma_{H,t}{}^{\nu-1} \mathrm{d}j \\ &+ \int_{j\in\mathcal{E}_t} \left[\frac{w_t}{\Xi}\right]^{1-\varepsilon\nu} (A_t z_t(j))^{\nu(\varepsilon-1)}\, n_{H,t}^{\varsigma-1}(j)^{1-\nu} \left[\frac{\alpha}{1-\alpha}\frac{w_t}{r_t^k}\right]^{\alpha(\varepsilon\nu-1)} \left(\Gamma_{H,t} + e^{\xi_t}\,\Gamma_{H,t}^{E*}\right)^{\nu-1} \mathrm{d}j \end{aligned}$$

$$\begin{aligned} P_{H,t}{}^{1-\varepsilon} = \left[\frac{w_t}{\Xi}\right]^{1-\varepsilon\nu} \left[\frac{\alpha}{1-\alpha}\frac{w_t}{r_t^k}\right]^{\alpha(\varepsilon\nu-1)} \Bigg[&\int_{j\in\{\mathcal{L}_t,\mathcal{M}_t\}} (A_t z_t(j))^{\nu(\varepsilon-1)}\, n_{H,t}^{\varsigma-1}(j)^{1-\nu} \Gamma_{H,t}{}^{\nu-1} \mathrm{d}j \\ &+ \int_{j\in\mathcal{E}_t} (A_t z_t(j))^{\nu(\varepsilon-1)}\, n_{H,t}^{\varsigma-1}(j)^{1-\nu} \left(\Gamma_{H,t} + e^{\xi_t}\,\Gamma_{H,t}^{E*}\right)^{\nu-1} \mathrm{d}j\Bigg] \end{aligned}$$

On the other hand, the price index for the Foreign-exported bundle sold to Home retailers $p_{F,t}^E$ is just the price of intermediate varieties from Foreign exporters $j \in \mathcal{E}_t^*$. It can be expressed as

$$p_{F,t}^E = \left[\int_{j\in\mathcal{E}_t^*} p_{F,t}^E(j)^{1-\varepsilon} \mathrm{d}j\right]^{\frac{1}{1-\varepsilon}} \tag{A.14}$$

By substituting the Foreign-equivalent of the labour demand (A.10) into the Foreign-equivalent intermediate variety price (A.3), the price index of this Foreign-exported bundle is:

$$p_{F,t}^{E\;1-\varepsilon} = \int_{j\in\mathcal{E}_t^*} Q_t{}^{1-\varepsilon} \left[\frac{w_t^*}{\Xi}\right]^{1-\varepsilon\nu} \left(A_t^* z_t^*(j)\right)^{\nu(\varepsilon-1)} (n_{F,t}^{E*}(j))^{1-\nu} \left[\frac{\alpha}{1-\alpha}\frac{w_t^*}{r_t^{k*}}\right]^{\alpha(\varepsilon\nu-1)} \left[\Gamma_{F,t}^{H*} + e^{\xi_t}\Gamma_{F,t}^{E}\right]^{\nu-1} \mathrm{d}j,$$

where $\Gamma_{F,t}^{H*} = p_{F,t}^{H*\,\varepsilon} Y_{F,t}^*$ and $\Gamma_{F,t} = \left(\frac{p_{F,t}^E}{Q_t}\right)^{\varepsilon} Y_{F,t}^E$.

Finally, the price index of the Foreign-developed bundle is just the price of intermediate varieties produced in Foreign-owned Home plants $j \in \mathcal{M}_t^*$:

$$p_{F,t}^M = \left[\int_{j\in\mathcal{M}_t^*} p_{F,t}^M(j)^{1-\varepsilon} \mathrm{d}j\right]^{\frac{1}{1-\varepsilon}}$$

By substituting the Foreign-equivalent of labour demand (A.12) into the Foreign-equivalent intermediate variety price (A.3), the price index can be expressed as:

$$p_{F,t}^{M\;1-\varepsilon} = \int_{j\in\mathcal{M}_j^*} \left[\frac{w_t}{\Xi}\right]^{1-\varepsilon\nu} \left(A_t^* z_t^*(j)\right)^{\nu(\varepsilon-1)} n_{F,t}^{\varsigma_{-1}*}(j)^{1-\nu} \left[\frac{\alpha}{1-\alpha}\frac{w_t}{r_t^k}\right]^{\alpha(\varepsilon\nu-1)} \Gamma_{F,t}^{M\;\nu-1} \mathrm{d}j, \tag{A.15}$$

where $\Gamma_{F,t}^M = p_{F,t}^{M\;\varepsilon} Y_{F,t}^M$

B.4 Intermediate Firms' Thresholds

B.4.1 Thresholds between staying local and exporting

There is a threshold $\tilde{z}_t^d$ such that, for a firm with export status $\varsigma_{t-1} = d = \{L, E\}$ in the previous period, the value function of being a local firm will equal the value function of being an exporter:

$$V_t^L(n_{H,t}^d(j), \tilde{z}_t^d, d) = V_t^E(n_{H,t}^d(j), \tilde{z}_t^d, d)$$

Expanding the two value functions, the threshold becomes

$$\begin{aligned} p_{H,t}^L(j)y_{H,t}^L(j) - w_t l_{H,t}^L(j) - r_t^k k_{H,t}^L(j) - p_t^s s_{H,t}^L(j) + \mathbb{E}_t M_{t,t+1} V_{t+1}(n_{H,t+1}^l(j), z_{t+1}(j), L) \\ = p_{H,t}^E(j)y_{H,t}^E(j) + Q_t p_{H,t}^{E*}(j) y_{H,t}^{E*}(j) - w_t l_{H,t}^E(j) - r_t^k k_{H,t}^E(j) - p_t^s s_{H,t}^E(j) \\ - Q_t w_t^* f^E(d) + \mathbb{E}_t M_{t,t+1} V_{t+1}(n_{H,t+1}^E(j), z_{t+1}(j), E) \end{aligned}$$

Rearranging the terms:

$$\begin{aligned} &Q_t w_t^* f^E(d) + p_t^s \left(n_{H,t+1}^E(j) - n_{H,t+1}^L(j)\right) \\ &\quad = \mathbb{E}_t M_{t,t+1} \left[V_{t+1}(n_{H,t+1}^E(j), z_{t+1}(j), E) - V_{t+1}(n_{H,t+1}^L(j), z_{t+1}(j), L)\right] \\ &\qquad + p_{H,t}^E(j)y_{H,t}^E(j) + Q_t p_{H,t}^{E*}(j) y_{H,t}^{E*}(j) - p_{H,t}^L(j) y_{H,t}^L(j) \\ &\qquad - w_t \left[l_{H,t}^E(j) - l_{H,t}^L(j)\right] - r_t^k \left[k_{H,t}^E(j) - k_{H,t}^L(j)\right] \end{aligned} \tag{A.16}$$

By substituting the optimal prices (A.2) into the equation, along with the demand functions for the intermediate varieties (18) and (19), the term on the right hand side $p^E_{H,t}(j)y^E_{H,t}(j) + Q_t p^{E*}_{H,t}(j)y^{E*}_{H,t}(j) - p^L_{H,t}(j)y^L_{H,t}(j) - w_t\left[l^E_{H,t}(j) - l^L_{H,t}(j)\right] - r^k_t\left[k^E_{H,t}(j) - k^L_{H,t}(j)\right]$ becomes

$$
\begin{aligned}
&= \left[\frac{\varepsilon}{\varepsilon-1}\frac{w_t l^E_{H,t}(j)}{(1-\chi)(1-\alpha)\,A_t\,\tilde{z}^d_t\,n^d_{H,t}(j)^{\chi}\left(k^E_{H,t}(j)^{\alpha} l^E_{H,t}(j)^{1-\alpha}\right)^{1-\chi}}\right]^{1-\varepsilon}\frac{Y_{H,t}}{p_{H,t}{}^{-\varepsilon}} \\
&\quad + \left[\frac{\varepsilon}{\varepsilon-1}\frac{w_t l^E_{H,t}(j)}{(1-\chi)(1-\alpha)\,A_t\,\tilde{z}^d_t\,n^d_{H,t}(j)^{\chi}\left(k^E_{H,t}(j)^{\alpha} l^E_{H,t}(j)^{1-\alpha}\right)^{1-\chi}}\right]^{1-\varepsilon}\frac{Y^{E*}_{H,t}}{\left[Q_t p^{E*}_{H,t}\right]^{-\varepsilon}} \\
&\quad - \left[\frac{\varepsilon}{\varepsilon-1}\frac{w_t l^L_{H,t}(j)}{(1-\chi)(1-\alpha)\,A_t\,\tilde{z}^d_t\,n^d_{H,t}(j)^{\chi}\left(k^L_{H,t}(j)^{\alpha} l^L_{H,t}(j)^{1-\alpha}\right)^{1-\chi}}\right]^{1-\varepsilon}\frac{Y_{H,t}}{p_{H,t}{}^{-\varepsilon}} \\
&\quad - w_t\left[l^E_{H,t}(j) - l^L_{H,t}(j)\right] - r^k_t\left[k^E_{H,t}(j) - k^L_{H,t}(j)\right]
\end{aligned}
$$

Substituting in the labour demands for exporters (A.10) and local firms (A.9), as well as the capital demand (A.5), this term is now

$$
\begin{aligned}
&= \left[\frac{w_t}{\Xi}\right]^{1-\varepsilon\nu}\left(A_t\tilde{z}^d_t\right)^{(\varepsilon-1)\nu} n^d_{H,t}(j)^{1-\nu}\left[\frac{\alpha}{1-\alpha}\frac{w_t}{r^k_t}\right]^{\alpha(\varepsilon\nu-1)} \\
&\quad \times\left[\left[\Gamma_{H,t} + e^{\xi_t}\Gamma^{E*}_{H,t}\right]^{\nu-1}\left[\Gamma_{H,t} + \Gamma^{E*}_{H,t}\right] - \Gamma_{H,t}{}^{\nu} - \frac{\Xi}{1-\alpha}\left[(\Gamma_{H,t} + e^{\xi_t}\Gamma^{E*}_{H,t})^{\nu} - \Gamma_{H,t}{}^{\nu}\right]\right]
\end{aligned}
$$

Plugging this back into the threshold condition (B.4.1), the threshold now becomes

$$
\begin{aligned}
&Q_t w^*_t f^E(d) + p^s_t\left(n^E_{H,t+1}(j) - n^L_{H,t+1}(j)\right) \\
&\quad = \left[\frac{w_t}{\Xi}\right]^{1-\varepsilon\nu}\left(A_t\tilde{z}^d_t\right)^{(\varepsilon-1)\nu} n^d_{H,t}(j)^{1-\nu}\left[\frac{\alpha}{1-\alpha}\frac{w_t}{r^k_t}\right]^{\alpha(\varepsilon\nu-1)} \\
&\qquad \times\left(\left[\Gamma_{H,t} + \Gamma^{E*}_{H,t}\right]\left[\Gamma_{H,t} + e^{\xi_t}\Gamma^{E*}_{H,t}\right]^{\nu-1} - (\Gamma_{H,t})^{\nu} - \frac{\Xi}{1-\alpha}\left[\left(\Gamma_{H,t} + e^{\xi_t}\Gamma^{E*}_{H,t}\right)^{\nu} - (\Gamma_{H,t})^{\nu}\right]\right) \\
&\quad + \mathbb{E}_t M_{t,t+1}\left[V_{t+1}(n^E_{H,t+1}(j), z_{t+1}(j), E) - V_{t+1}(n^L_{H,t+1}(j), z_{t+1}(j), L)\right]
\end{aligned}
\tag{A.17}
$$

B.4.2 Threshold between exporting and being a multinational firm

There is a threshold $\hat{z}_t^f$ such that for a firm with status $\varsigma_{t-1} = f = \{E, M\}$ in the previous period, the value from exporting equals the value from being a multinational firm:

$$V_t^E(n_{H,t}^f, \hat{z}_t^f, f) = V_t^M(n_{H,t}^f, \hat{z}_t^f, f)$$

Expanding the value functions, the threshold becomes

$$\begin{aligned}
&p_{H,t}^E(j)y_{H,t}^E(j) + Q_t p_{H,t}^{E*}(j)y_{H,t}^{E*}(j) - w_t l_{H,t}^E(j) - r_t^k k_{H,t}^E(j) - p_t^s s_{H,t}^L(j) \\
&\qquad - Q_t w_t^* f^E(f) + \mathbb{E}_t M_{t,t+1} V_{t+1}(n_{H,t+1}^E(j), z_{t+1}(j), E) \\
&= p_{H,t}^M(j)y_{H,t}^M(j) - w_t l_{H,t}^M(j) - r_t^k k_{H,t}^M(j) - p_t^s s_{H,t}^M(j) \\
&\qquad + Q_t p_{H,t}^{M*}(j)y_{H,t}^{M*}(j) - Q_t w_t^* l_{F,t}^M(j) - Q_t r_t^{k*} k_{F,t}^M(j) \\
&\qquad - Q_t w_t^* f^M(f) + \mathbb{E}_t M_{t,t+1} V_{t+1}(n_{H,t+1}^M(j), z_{t+1}(j), M)
\end{aligned} \tag{A.18}$$

By plugging in the optimal prices (A.2) and (A.4) functions for the intermediate varieties (20) and (19), the term $p_{H,t}^M(j)y_{H,t}^M(j) + Q_t p_{H,t}^{M*}(j)y_{H,t}^{M*}(j) - p_{H,t}^E(j)y_{H,t}^E(j) - Q_t p_{H,t}^{E*}(j)y_{H,t}^{E*}(j) - w_t \left[l_{H,t}^M(j) - l_{H,t}^E(j)\right] - Q_t w_t^* l_{F,t}^M(j) - r_t^k \left[k_{H,t}^M(j) - k_{H,t}^E(j)\right] - Q_t r_t^{k*} k_{F,t}^M(j)$ becomes

$$\begin{aligned}
&= \left[\frac{\varepsilon}{\varepsilon - 1} \frac{w_t l_{H,t}^M(j)}{(1-\chi)(1-\alpha)A_t \hat{z}_t^f n_{H,t}^d(j)^\chi \left(k_{H,t}^M(j)^\alpha l_{H,t}^M(j)^{1-\alpha}\right)^{1-\chi}}\right]^{1-\varepsilon} \frac{Y_{H,t}}{p_{H,t}{}^{-\varepsilon}} \\
&\quad + \left[\frac{\varepsilon}{\varepsilon - 1} \frac{w_t^* l_{F,t}^M(j)}{(1-\chi)(1-\alpha)A_t \hat{z}_t^f n_{H,t}^d(j)^\chi \left(k_{F,t}^M(j)^\alpha l_{F,t}^M(j)^{1-\alpha}\right)^{1-\chi}}\right]^{1-\varepsilon} \frac{Y_{H,t}^{M*}}{p_{H,t}^{M*-\varepsilon}} Q_t \\
&\quad - \left[\frac{\varepsilon}{\varepsilon - 1} \frac{w_t l_{H,t}^E(j)}{(1-\chi)(1-\alpha)A_t \hat{z}_t^f n_{H,t}^d(j)^\chi \left(k_{H,t}^E(j)^\alpha l_{H,t}^E(j)^{1-\alpha}\right)^{1-\chi}}\right]^{1-\varepsilon} \left[p_{H,t}{}^\varepsilon Y_{H,t} + \left(Q_t p_{H,t}^{E*}\right)^\varepsilon Y_{H,t}^{E*}\right] \\
&\quad - w_t \left[l_{H,t}^M(j) - l_{H,t}^E(j)\right] - Q_t w_t^* l_{F,t}^M(j) - r_t^k \left[k_{H,t}^M(j) - k_{H,t}^E(j)\right] - Q_t r_t^{k*} k_{F,t}^M(j)
\end{aligned}$$

Substituting in the labour demands (A.10), (A.11), and (A.12), as well as the physical capital demand (A.5),

$$
\begin{aligned}
&= \left[\frac{w_t}{\Xi}\right]^{1-\varepsilon\nu} \left(A_t \hat{z}_t^f\right)^{(\varepsilon-1)\nu} {n_{H,t}^f}^{1-\nu} \left[\frac{\alpha}{1-\alpha}\frac{w_t}{r_t^k}\right]^{\alpha(\varepsilon\nu-1)} \\
&\qquad \times \left[(\Gamma_{H,t})^\nu - \left[\Gamma_{H,t} + \Gamma_{H,t}^{E*}\right] \left[\Gamma_{H,t} + e^{\xi_t}\Gamma_{H,t}^{E*}\right]^{\nu-1} - \frac{\Xi}{1-\alpha}\left[{\Gamma_{H,t}}^\nu - (\Gamma_{H,t} + e^{\xi_t}\Gamma_{H,t}^{E*})^\nu\right] \right] \\
&\quad + \left[\frac{w_t^*}{\Xi}\right]^{1-\varepsilon\nu} Q_t \left[1 - \frac{\Xi}{1-\alpha}\right] \left(A_t \hat{z}_t^f\right)^{(\varepsilon-1)\nu} {n_{H,t}^f}^{1-\nu} \left[\frac{\alpha}{1-\alpha}\frac{w_t^*}{r_t^{k*}}\right]^{\alpha(\varepsilon\nu-1)} {\Gamma_{H,t}^{M*}}^\nu
\end{aligned}
$$

Plugging this back into the threshold (A.18), it can now be rewritten

$$
\begin{aligned}
&Q_t w_t^* [f^M(f) - f^E(f)] + p_t^s \left(n_{H,t+1}^M(j) - n_{H,t+1}^E(j)\right) \\
&= \left(A_t \hat{z}_t^f\right)^{(\varepsilon-1)\nu} {n_{H,t}^f}^{1-\nu} \left[\frac{\alpha}{1-\alpha}\right]^{\alpha(\varepsilon\nu-1)} \\
&\quad \times \Bigg(\left[\frac{w_t}{\Xi}\right]^{1-\varepsilon\nu} \left[\frac{w_t}{r_t^k}\right]^{\alpha(\varepsilon\nu-1)} \left[{\Gamma_{H,t}}^\nu - \left(\Gamma_{H,t} + \Gamma_{H,t}^{E*}\right)\left(\Gamma_{H,t} + e^{\xi_t}\Gamma_{H,t}^{E*}\right)^{\nu-1} - \frac{\Xi\left[{\Gamma_{H,t}}^\nu - (\Gamma_{H,t} + e^{\xi_t}\Gamma_{H,t}^{E*})^\nu\right]}{1-\alpha} \right] \\
&\qquad + \left[\frac{w_t^*}{\Xi}\right]^{1-\varepsilon\nu} \left[\frac{w_t^*}{r_t^{k*}}\right]^{\alpha(\varepsilon\nu-1)} Q_t \left[1 - \frac{\Xi}{1-\alpha}\right] {\Gamma_{H,t}^{M*}}^\nu \Bigg) \\
&\quad + \mathbb{E}_t M_{t,t+1}\left[V_{t+1}(n_{H,t+1}^M(j), z_{t+1}(j), M) - V_{t+1}(n_{H,t+1}^E(j), z_{t+1}(j), E)\right]
\end{aligned} \tag{A.19}
$$

C Model Equations

C.1 Household

The representative household chooses $C_t, L_t, B_{H,t+1}, B_{F,t+1}, I_t, S_t$, to maximize expected lifetime utility

$$\max_{\{C_t,L_t,I_t,S_t,B_{H,t+1},B_{F,t+1}\}} \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \frac{\left[(C_t - \mathrm{b}\, C_{t-1}) - \frac{\psi}{1+\mu} L_t^{1+\mu}\right]^{1-\sigma}}{1-\sigma}, \tag{C.1}$$

subject to

$$\begin{aligned} C_t + &\left[1 + \frac{\kappa}{2}(\frac{S_t}{S_{t-1}} - 1)^2\right] S_t + I_t + \frac{B_{H,t+1}}{P_t} + \frac{\epsilon_t B_{F,t+1}}{P_t} \\ &= w_t L_t + r_t^k K_t + \frac{R_{t-1}}{P_t} B_{H,t} + \frac{\epsilon_t R_{t-1}^*}{P_t} B_{F,t} + T_t + p_t^s S_t + \int \frac{\Pi_{Y,t}(i)}{P_t} \mathrm{d}i + \int \frac{\Pi_t(j)}{P_t} \mathrm{d}j, \end{aligned} \tag{C.2}$$

$$K_{t+1} = (1-\delta_k) K_t + \left[1 - \frac{\kappa}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2\right] I_t, \tag{C.3}$$

Optimality conditions are:

$$\left[\frac{C_t - bC_{t-1}}{1-b} - \frac{\psi}{1+\mu} L_t^{1+\mu}\right]^{-\gamma} = \lambda_t, \tag{C.4}$$

where $\lambda_t = U_{C,t}$ is the Lagrange multiplier attached to the Household budget constraint.

$$\mathbb{E}_t \left[M_{t,t+1} \frac{R_t P_t}{P_{t+1}}\right] = 1; \tag{C.5}$$

$$\mathbb{E}_t \left[M_{t,t+1} \frac{R_t^* \epsilon_{t+1} P_t}{\epsilon_t P_{t+1}}\right] = 1, \tag{C.6}$$

where $M_{t,t+1} = \beta \frac{\lambda_{t+1}}{\lambda_t}$ is the stochastic discount factor.

$$\left[C_t - \frac{\psi}{1+\mu} L_t^{1+\mu}\right]^{-\gamma} \psi L_t^{\mu} = w_t \tag{C.7}$$

$$1 = q_t \left[1 - \frac{\kappa}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2\right] - q_t \kappa \left(\frac{I_t}{I_{t-1}} - 1\right) \frac{I_t}{I_{t-1}} + \kappa \mathbb{E}_t M_{t,t+1} q_{t+1} \left(\frac{I_{t+1}}{I_t} - 1\right) \left(\frac{I_{t+1}}{I_t}\right)^2 \tag{C.8}$$

$$q_t = \mathbb{E}_t \left[r^k_{t+1} + q_{t+1} \left(1 - \delta_k\right) \right], \tag{C.9}$$

where q_t is the Tobin's Q marginal ratio.

$$p^s_t = 1 + \frac{\kappa}{2} \left(\frac{S_t}{S_{t-1}} - 1 \right)^2 + \kappa \left(\frac{S_t}{S_{t-1}} - 1 \right) \frac{S_t}{S_{t-1}} - \mathbb{E}_t M_{t,t+1} \kappa \left(\frac{S_{t+1}}{S_t} - 1 \right) \left(\frac{S_{t+1}}{S_t} \right)^2 \tag{C.10}$$

C.2 Retailers

Retailers choose $Y_t(i), P_t(i), Y_{H,t}(i), Y^E_{F,t}(i), Y^M_{F.t}(i)$

$$\max_{\substack{Y_t(i),P_t(i), \\ Y_{H,t}(i),Y^E_{F,t}(i),Y^M_{F.t}(i)}} E_t \sum_{t=0}^{\infty} M_{t,t+1} \frac{\Pi_{Y,t}(i)}{P_t} \tag{C.11}$$

subject to

$$\Pi_{Y,t}(i) = P_t(i) Y_t(i) - P_{H,t} Y_{H,t}(i) - P_{F,t} Y_{F,t}(i) - AC_t(i), \tag{C.12}$$

$$AC_t(i) = \frac{\rho_P}{2} \left(\frac{P_t(i)}{P_{t-1}(i)} - 1 \right)^2 Y_t \tag{C.13}$$

$$Y_t(i) = \left[\omega^{\frac{1}{\theta}} Y_{H,t}(i)^{\frac{\theta-1}{\theta}} + (1-\omega)^{\frac{1}{\theta}} \; Y_{F,t}(i)^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}, \tag{C.14}$$

$$Y_{F,t}(i) = \left[\nu^{\frac{1}{\eta}} Y^E_{F,t}(i)^{\frac{\eta-1}{\eta}} + (1-\nu)^{\frac{1}{\eta}} \; Y^M_{F,t}(i)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}, \tag{C.15}$$

$$Y_{H,t} = \left[\int_{j \in [0,1]} y_{H,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} \mathrm{d}j \right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{C.16}$$

$$Y^M_{F,t} = \left[\int_{j \in \mathcal{M}^*_t} y^M_{F,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} \mathrm{d}j \right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{C.17}$$

$$Y^E_{F,t} = \left[\int_{j \in \mathcal{E}^*_t} y^E_{F,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} \mathrm{d}j \right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{C.18}$$

Optimality conditions are:

$$\rho_P \left(\pi^P_t - 1\right) \pi^P_t = \phi \left[\frac{MC_t}{P_t} - \frac{\phi - 1}{\phi} \right] + \rho_P \mathbb{E}_t M_{t,t+1} \left(\pi^P_{t+1} - 1\right) \pi^P_{t+1} \frac{Y_{t+1}}{Y_t}, \tag{C.19}$$

where $\pi^P_t = \frac{P_t}{P_{t-1}}$.

$$MC_t = \left[\omega\left(P_{H,t}\right)^{1-\theta} + (1-\omega)\left(P_{F,t}\right)^{1-\theta}\right]^{\frac{1}{1-\theta}} \tag{C.20}$$

$$p_{H,t} = \left[\int_{j\in[0,1]} p_{H,t}(j)^{1-\varepsilon}\mathrm{d}j\right]^{\frac{1}{1-\varepsilon}} \tag{C.21}$$

$$y^{\varsigma}_{H,t}(j) = \left[\frac{P^{\varsigma}_{H,t}(j)}{P_{H,t}}\right]^{-\varepsilon} Y_{H,t} = \omega\left[\frac{P^{\varsigma}_{H,t}(j)}{P_{H,t}}\right]^{-\varepsilon}\left[\frac{P_{H,t}}{MC_t}\right]^{-\theta} Y_t(i), \quad \varsigma \in \{L, E, M\}; \tag{C.22}$$

$$p^{E}_{F,t} = \left[\int_{j\in\mathcal{E}^*_t} p^{E}_{F,t}(j)^{1-\varepsilon}\mathrm{d}j\right]^{\frac{1}{1-\varepsilon}} \tag{C.23}$$

$$y^{E}_{F,t}(j) = \left[\frac{P^{E}_{F,t}(j)}{P^{E}_{F,t}}\right]^{-\varepsilon} Y^{E}_{F,t} = \nu(1-\omega)\left[\frac{P^{E}_{F,t}(j)}{P^{E}_{F,t}}\right]^{-\varepsilon}\left[\frac{e^{\tau_t}\, P^{E}_{F,t}}{P_{F,t}}\right]^{-\eta}\left[\frac{P_{F,t}}{MC_t}\right]^{-\theta} Y_t(i), \tag{C.24}$$

$$p^{M}_{F,t} = \left[\int_{j\in\mathcal{M}^*_t} p^{M}_{F,t}(j)^{1-\varepsilon}\mathrm{d}j\right]^{\frac{1}{1-\varepsilon}} \tag{C.25}$$

$$y^{M}_{F,t}(j) = \left[\frac{P^{M}_{F,t}(j)}{P^{M}_{F,t}}\right]^{-\varepsilon} Y^{M}_{F,t} = (1-\nu)(1-\omega)\left[\frac{P^{M}_{F,t}(j)}{P^{M}_{F,t}}\right]^{-\varepsilon}\left[\frac{P^{M}_{F,t}}{P_{F,t}}\right]^{-\eta}\left[\frac{P_{F,t}}{MC_t}\right]^{-\theta} Y_t(i) \tag{C.26}$$

C.3 Intermediate Variety Firms

Let $V_t(n^{\varsigma-1}_{H,t}, z_t, \varsigma_{t-1}; A_t, e^{\xi_t})$ be the optimal value of an intermediate variety firm with individual states $(n^{\varsigma-1}_{H,t}, z_t, \varsigma_{t-1})$ and aggregate states (A_t, e^{ξ_t}). $V_t(n^{\varsigma-1}_{H,t}, z_t, \varsigma_{t-1}; A_t, e^{\xi_t})$ solves the following Bellman equation

$$\begin{aligned} V_t\left(n^{\varsigma-1}_{H,t}, z_t, \varsigma_{t-1}; A_t, e^{\xi_t}\right) = & \max_{\substack{\varsigma_t, s_{H,t}, n_{H,t+1},\\ l_{H,t}, l^{M}_{F,t}, k_{H,t}, k^{M}_{F,t}\\ p_{H,t}, p^*_{H,t},\\ y_{H,t}, y^*_{H,t}}} \pi_t(j) + \mathbb{E}_t M_{t,t+1} V_{t+1}\left(n_{H,t+1}, z_{t+1}, \varsigma_t; A_{t+1}, e^{\xi_{t+1}}\right) \\ & - \begin{cases} 0 & \text{if } \varsigma_t(j) = L \\ Q_t w^*_t f^{E}(\varsigma_{t-1}) & \text{if } \varsigma_t(j) = E \\ Q_t w^*_t f^{M}(\varsigma_{t-1}) & \text{if } \varsigma_t(j) = M \end{cases} \end{aligned} \tag{C.27}$$

subject to

$$\pi_t^L(j) = p_{H,t}^L(j) y_{H,t}^L(j) - w_t l_{H,t}^L(j) - r_t^k k_{H,t}^L(j) - p_t^s s_{H,t}^L(j), \quad j \in \mathcal{L}_t; \tag{C.28a}$$

$$\pi_t^E(j) = p_{H,t}^E(j) y_{H,t}^E(j) + Q_t p_{H,t}^*(j) y_{H,t}^{E*}(j) - w_t l_{H,t}^E(j) - r_t^k k_{H,t}^E(j) - p_t^s s_{H,t}^E(j), \quad j \in \mathcal{E}_t; \tag{C.28b}$$

$$\begin{aligned} \pi_t^M(j) = {} & p_{H,t}^M(j) y_{H,t}^M(j) - w_t l_{H,t}^M(j) - r_t^k k_{H,t}^M(j) - p_t^s s_{H,t}^M(j) \\ & + Q_t p_{H,t}^{M*}(j) y_{H,t}^{M*}(j) - Q_t w_t^* l_{F,t}^M(j) - Q_t r_t^{k*} k_{F,t}^M(j), \quad j \in \mathcal{M}_t \end{aligned} \tag{C.28c}$$

$$y_{H,t}(j) + E_t(j)\, e^{\xi_t}\, y_{H,t}^{E*}(j) = A_t z_t(j)\, n_{H,t}^{\varsigma-1}(j)^{\chi} \left(k_{H,t}(j)^{\alpha} l_{H,t}(j)^{1-\alpha}\right)^{1-\chi}, \quad j \in \{\mathcal{L}_t, \mathcal{E}_t, \mathcal{M}_t\}; \tag{C.29a}$$

$$y_{H,t}^{M*}(j) = A_t z_t(j) n_{H,t}^{\varsigma-1}(j)^{\chi} \left(k_{F,t}^M(j)^{\alpha} l_{F,t}^M(j)^{1-\alpha}\right)^{1-\chi}, \quad j \in \mathcal{M}_t \tag{C.29b}$$

$$n_{H,t+1}(j) = (1-\delta_n) n_{H,t}^{\varsigma-1}(j) + s_{H,t}(j), \quad j \in \{\mathcal{L}_t, \mathcal{E}_t, \mathcal{M}_t\} \tag{C.30}$$

Optimality Conditions are:

$$p_{H,t}(j) = \frac{\varepsilon}{\varepsilon-1} \frac{w_t l_{H,t}(j)}{(1-\chi)(1-\alpha) A_t z_t(j) n_{H,t}^{\varsigma-1}(j)^{\chi} \left(k_{H,t}(j)^{\alpha} l_{H,t}(j)^{1-\alpha}\right)^{1-\chi}}, \quad j \in \{\mathcal{L}_t, \mathcal{E}_t, \mathcal{M}_t\} \tag{C.31}$$

$$p_{H,t}^{E*}(j) = \frac{1}{Q_t} \frac{\varepsilon}{\varepsilon-1} \frac{w_t l_{H,t}(j)}{(1-\chi)(1-\alpha) A_t z_t(j) n_{H,t}^{\varsigma-1}(j)^{1-\chi} \left(k_{H,t}^E(j)^{\alpha} l_{H,t}^E(j)^{1-\alpha}\right)^{1-\chi}}, \quad j \in \mathcal{E}_t \tag{C.32}$$

$$p_{H,t}^{M*}(j) = \frac{1}{Q_t} \frac{\varepsilon}{\varepsilon-1} \frac{Q_t w_t^* l_{F,t}^M(j)}{(1-\chi)(1-\alpha) A_t z_t(j) n_{H,t}^{\varsigma-1}(j)^{\chi} \left(k_{F,t}^M(j)^{\alpha} l_{F,t}^M(j)^{1-\alpha}\right)^{1-\chi}}, \quad j \in \mathcal{M}_t \tag{C.33}$$

$$l_{H,t}(j) = \left[\frac{w_t}{\Xi}\right]^{-\varepsilon\nu} (A_t z_t(j))^{(\varepsilon-1)\nu}\, n_{H,t}^{\varsigma-1}(j)^{1-\nu} \left[\frac{\alpha}{1-\alpha} \frac{w_t}{r_t^k}\right]^{\alpha(\varepsilon\nu-1)} \left[\Gamma_{H,t} + E_t(j) e^{\xi_t} \Gamma_{H,t}^{E*}\right]^{\nu}, \quad j \in \{\mathcal{L}_t, \mathcal{E}_t, \mathcal{M}_t\} \tag{C.34a}$$

$$l_{F,t}^M(j) = \left[\frac{w_t^*}{\Xi}\right]^{-\varepsilon\nu} (A_t z_t(j))^{(\varepsilon-1)\nu}\, n_{H,t}^{\varsigma-1}(j)^{1-\nu} \left[\frac{\alpha}{1-\alpha} \frac{w_t^*}{r_t^{k*}}\right]^{\alpha(\varepsilon\nu-1)} \Gamma_{H,t}^{M*}(j)^{\nu}, \quad j \in \mathcal{M}_t \tag{C.34b}$$

$$k_{H,t}(j) = \frac{\alpha}{1-\alpha}\frac{w_t}{r_t^k} l_{H,t}(j), \qquad j \in \{\mathcal{L}_t, \mathcal{E}_t, \mathcal{M}_t\} \tag{C.35a}$$

$$k_{F,t}^M(j) = \frac{\alpha}{1-\alpha}\frac{w_t^*}{r_t^{k*}} l_{F,t}^M(j), \qquad j \in \mathcal{M}_t \tag{C.35b}$$

$$p_t^s = \mathbb{E}_t M_{t,t+1}\left[(1-\delta_n)p_{t+1}^s + \frac{\chi}{(1-\chi)(1-\alpha)}\frac{w_{t+1} l_{H,t+1}(j)}{n_{H,t+1}(j)}\right], \quad j \in \{\mathcal{L}_t, \mathcal{E}_t\} \tag{C.36a}$$

$$p_t^s = \mathbb{E}_t M_{t,t+1}\left[(1-\delta_n)p_{t+1}^s + \frac{\chi}{(1-\chi)(1-\alpha)}\left(\frac{w_{t+1} l_{H,t+1}(j)}{n_{H,t+1}(j)} + \frac{w_{t+1}^* Q_{t+1} l_{F,t+1}(j)}{n_{H,t+1}(j)}\right)\right], \quad j \in \mathcal{M}_t \tag{C.36b}$$

Thresholds between staying local and exporting:
(i) $\tilde{z}_t^L(j)$: threshold for a firm who is a local firm in the previous period

$$\begin{aligned}
&Q_t w_t^* f^E(L) + p_t^s\left(n_{H,t+1}^E(j) - n_{H,t+1}^L(j)\right) \\
&\quad = \left[\frac{w_t}{\Xi}\right]^{1-\varepsilon\nu}\left(A_t \tilde{z}_t^L(j)\right)^{(\varepsilon-1)\nu}(n_{H,t}^{L_{-1}}(j))^{1-\nu}\left[\frac{\alpha}{1-\alpha}\frac{w_t}{r_t^k}\right]^{\alpha(\varepsilon\nu-1)} \\
&\qquad \times\left[\left[\Gamma_{H,t} + \Gamma_{H,t}^{E*}\right]\left[\Gamma_{H,t} + e^{\xi_t}\Gamma_{H,t}^{E*}\right]^{\nu-1} - \Gamma_{H,t}{}^{\nu} - \frac{\Xi}{1-\alpha}\left[\left(\Gamma_{H,t} + e^{\xi_t}\Gamma_{H,t}^{E*}\right)^{\nu} - \Gamma_{H,t}{}^{\nu}\right]\right] \\
&\quad + \mathbb{E}_t M_{t,t+1}\left[V_{t+1}(n_{H,t+1}^E(j), z_{t+1}(j), E) - V_{t+1}(n_{H,t+1}^L(j), z_{t+1}(j), L)\right];
\end{aligned} \tag{C.37a}$$

(ii) $\tilde{z}_t^E(j)$: threshold for a firm who is an exporter in the previous period

$$\begin{aligned}
&Q_t w_t^* f^E(E) + p_t^s\left(n_{H,t+1}^E(j) - n_{H,t+1}^L(j)\right) \\
&\quad = \left[\frac{w_t}{\Xi}\right]^{1-\varepsilon\nu}\left(A_t \tilde{z}_t^E(j)\right)^{(\varepsilon-1)\nu}(n_{H,t}^{E_{-1}}(j))^{1-\nu}\left[\frac{\alpha}{1-\alpha}\frac{w_t}{r_t^k}\right]^{\alpha(\varepsilon\nu-1)} \\
&\qquad \times\left[\left[\Gamma_{H,t} + \Gamma_{H,t}^{E*}\right]\left[\Gamma_{H,t} + e^{\xi_t}\Gamma_{H,t}^{E*}\right]^{\nu-1} - \Gamma_{H,t}{}^{\nu} - \frac{\Xi}{1-\alpha}\left[\left(\Gamma_{H,t} + e^{\xi_t}\Gamma_{H,t}^{E*}\right)^{\nu} - \Gamma_{H,t}{}^{\nu}\right]\right] \\
&\quad + \mathbb{E}_t M_{t,t+1}\left[V_{t+1}(n_{H,t+1}^E(j), z_{t+1}(j), E) - V_{t+1}(n_{H,t+1}^L(j), z_{t+1}(j), L)\right],
\end{aligned} \tag{C.37b}$$

Thresholds between exporting and operating as a multinational firm:

(i) $\hat{z}_t^E(j)$: threshold for a firm who is an exporter in the previous period

$$
\begin{aligned}
&Q_t w_t^* [f^M(E) - f^E(E)] + p_t^s \left(n_{H,t+1}^M(j) - n_{H,t+1}^E(j)\right) \\
&= \left(A_t \hat{z}_t^E(j)\right)^{(\varepsilon-1)\nu} n_{H,t}^{E_{-1}}(j)^{1-\nu} \left[\frac{\alpha}{1-\alpha}\right]^{\alpha(\varepsilon\nu-1)} \\
&\quad \times\Bigg(\left[\frac{w_t}{\Xi}\right]^{1-\varepsilon\nu} \left[\frac{w_t}{r_t^k}\right]^{\alpha(\varepsilon\nu-1)} \left[{\Gamma_{H,t}}^{\nu} - \left(\Gamma_{H,t} + \Gamma_{H,t}^{E*}\right)\left(\Gamma_{H,t} + e^{\xi_t}\Gamma_{H,t}^{E*}\right)^{\nu-1} - \frac{\Xi\left[{\Gamma_{H,t}}^{\nu} - (\Gamma_{H,t} + e^{\xi_t}\Gamma_{H,t}^{E*})^{\nu}\right]}{1-\alpha}\right] \\
&\qquad + \left[\frac{w_t^*}{\Xi}\right]^{1-\varepsilon\nu} \left[\frac{w_t^*}{r_t^{k*}}\right]^{\alpha(\varepsilon\nu-1)} Q_t \left[1 - \frac{\Xi}{1-\alpha}\right] {\Gamma_{H,t}^{M*}}^{\nu}\Bigg) \\
&\quad + \mathbb{E}_t M_{t,t+1}\left[V_{t+1}(n_{H,t+1}^M(j), z_{t+1}(j), M) - V_{t+1}(n_{H,t+1}^E(j), z_{t+1}(j), E)\right]
\end{aligned} \tag{C.38a}
$$

(ii) $\hat{z}_t^M(j)$: threshold for a firm who is a multinational firm at the beginning of the period

$$
\begin{aligned}
&Q_t w_t^* [f^M(M) - f^E(M)] + p_t^s \left(n_{H,t+1}^M(j) - n_{H,t+1}^E(j)\right) \\
&= \left(A_t \hat{z}_t^M(j)\right)^{(\varepsilon-1)\nu} n_{H,t}^{M_{-1}}(j)^{1-\nu} \left[\frac{\alpha}{1-\alpha}\right]^{\alpha(\varepsilon\nu-1)} \\
&\quad \times\Bigg(\left[\frac{w_t}{\Xi}\right]^{1-\varepsilon\nu} \left[\frac{w_t}{r_t^k}\right]^{\alpha(\varepsilon\nu-1)} \left[{\Gamma_{H,t}}^{\nu} - \left(\Gamma_{H,t} + \Gamma_{H,t}^{E*}\right)\left(\Gamma_{H,t} + e^{\xi_t}\Gamma_{H,t}^{E*}\right)^{\nu-1} - \frac{\Xi\left[{\Gamma_{H,t}}^{\nu} - (\Gamma_{H,t} + e^{\xi_t}\Gamma_{H,t}^{E*})^{\nu}\right]}{1-\alpha}\right] \\
&\qquad + \left[\frac{w_t^*}{\Xi}\right]^{1-\varepsilon\nu} \left[\frac{w_t^*}{r_t^{k*}}\right]^{\alpha(\varepsilon\nu-1)} Q_t \left[1 - \frac{\Xi}{1-\alpha}\right] {\Gamma_{H,t}^{M*}}^{\nu}\Bigg) \\
&\quad + \mathbb{E}_t M_{t,t+1}\left[V_{t+1}(n_{H,t+1}^M(j), z_{t+1}(j), M) - V_{t+1}(n_{H,t+1}^E(j), z_{t+1}(j), E)\right]
\end{aligned} \tag{C.38b}
$$

C.4 Taylor Rule

$$
R_t = (R_{t-1})^{\phi_R} \left[R\left(\frac{\pi_t}{\pi}\right)^{\phi_\pi}\left(\frac{Y_t}{Y_{t-1}}\right)^{\phi_Y}\right]^{1-\phi_R} \tag{C.39}
$$

C.5 Government

$$
\frac{e^{\tau_t} P_{F,t}^E}{P_t} Y_{F,t}^E = T_t \tag{C.40}
$$

C.6 Market Clearing

Law of motion for exporters:

$$|\mathcal{E}_t| = (1 - |\mathcal{E}_{t-1}| - |\mathcal{M}_{t-1}|) \times \mathbb{P}_{LE,t} + |\mathcal{E}_{t-1}| \times \mathbb{P}_{EE,t} + |\mathcal{M}_{t-1}| \times \mathbb{P}_{ME,t} \tag{C.41}$$

Law of motion for multinational firms:

$$|\mathcal{M}_t| = |\mathcal{E}_{t-1}| \times \mathbb{P}_{EM,t} + |\mathcal{M}_{t-1}| \times \mathbb{P}_{MM,t} \tag{C.42}$$

Transition Frequencies:

$$\mathbb{P}_{LL,t} = \mathbb{N}\left(\frac{\tilde{z}_t^L}{\sigma_z}\right); \qquad \mathbb{P}_{LE,t} = 1 - \mathbb{N}\left(\frac{\tilde{z}_t^L}{\sigma_z}\right) \tag{C.43a}$$

$$\mathbb{P}_{EL,t} = \mathbb{N}\left(\frac{\tilde{z}_t^E}{\sigma_z}\right); \qquad \mathbb{P}_{EE,t} = \mathbb{N}\left(\frac{\hat{z}_t^E}{\sigma_z}\right) - \mathbb{N}\left(\frac{\tilde{z}_t^E}{\sigma_z}\right); \qquad \mathbb{P}_{EM,t} = 1 - \mathbb{N}\left(\frac{\hat{z}_t^E}{\sigma_z}\right) \tag{C.43b}$$

$$\mathbb{P}_{ME,t} = \mathbb{N}\left(\frac{\hat{z}_t^M}{\sigma_z}\right); \qquad \mathbb{P}_{MM,t} = 1 - \mathbb{N}\left(\frac{\hat{z}_t^M}{\sigma_z}\right). \tag{C.43c}$$

Labour market clearing:

$$\begin{aligned} L_t = {} & (1 - |\mathcal{E}_t| - |\mathcal{M}_t|)\, l_{H,t}^L + |\mathcal{E}_t|\, l_{H,t}^E + |\mathcal{M}_t|\, l_{H,t}^M + |\mathcal{M}_t^*|\, l_{H,t}^{M*} \\ & + (1 - |\mathcal{E}_{t-1}^*| - |\mathcal{M}_{t-1}^*|)\, \mathbb{P}_{LE,t}^* f^E(L) + |\mathcal{E}_{t-1}^*|\, \mathbb{P}_{EE,t}^*\, f^E(E) \\ & + |\mathcal{E}_{t-1}^*|\, \mathbb{P}_{EM,t}^*\, f^M(E) + |\mathcal{M}_{t-1}^*|\, \mathbb{P}_{MM,t}^*\, f^M(M), \end{aligned} \tag{C.44}$$

Physical capital market clearing:

$$K_t = (1 - |\mathcal{E}_t| - |\mathcal{M}_t|)\, k_{H,t}^L + |\mathcal{E}_t|\, k_{H,t}^E + |\mathcal{M}_t|\, k_{H,t}^M + |\mathcal{M}_t^*|\, k_{H,t}^{M*}, \tag{C.45}$$

Technological capital market clearing:

$$N_{t+1} = (1 - |\mathcal{E}_t| - |\mathcal{M}_t|)\, n_{H,t+1}^L + |\mathcal{E}_t|\, n_{H,t+1}^E + |\mathcal{M}_t|\, n_{H,t+1}^M \tag{C.46}$$

Final goods market clearing:

$$Y_t = C_t + I_t + \left[1 + \frac{\kappa}{2}(\frac{S_t}{S_{t-1}} - 1)^2\right] S_t + \frac{\rho_P}{2}(\frac{P_t}{P_{t-1}} - 1)^2 Y_t \tag{C.47}$$

The bundle market clears:

$$\int_{i\in[0,1]} Y_{H,t}(i)\, \mathrm{d}i = Y_{H,t} \tag{C.48}$$

$$\int_{i\in[0,1]} Y^M_{H,t}(i)\, \mathrm{d}i = Y^M_{H,t} \tag{C.49}$$

$$\int_{i\in[0,1]} Y^E_{F,t}(i)\, \mathrm{d}i = Y^E_{F,t} \tag{C.50}$$

The intermediate variety market clears:

$$Y_{H,t} = \left[\int_{j\in[0,1]} y_{H,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} \mathrm{d}j\right]^{\frac{\varepsilon}{\varepsilon-1}}; \tag{C.51}$$

$$Y^E_{F,t} = \left[\int_{j\in\mathcal{E}^*_t} y^E_{F,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} \mathrm{d}j\right]^{\frac{\varepsilon}{\varepsilon-1}}; \tag{C.52}$$

$$Y^M_{F,t} = \left[\int_{j\in\mathcal{M}^*_t} y^M_{F,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} \mathrm{d}j\right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{C.53}$$

Bond market clearing:

$$\frac{B_{H,t}}{P_t} + \frac{B^*_{H,t}}{\epsilon_t P^*_t} = 0 \tag{C.54}$$